8. $f(x,y)=y\ln x$

(a) $\nabla f(x,y)=\left\langle f_x(x,y),f_y(x,y)\right\rangle=\langle y/x,\ln x\rangle$

(b) $\nabla f(1,-3)=\left\langle \frac{-3}{1},\ln 1\right\rangle=\langle -3,0\rangle$

(c) By Equation 9, $D_{\mathbf{u}} f(1,-3)=\nabla f(1,-3)\cdot \mathbf{u}=\langle -3,0\rangle\cdot \left\langle -\frac{4}{5},\frac{3}{5}\right\rangle=\frac{12}{5}$.

9. $f(x,y,z)=xe^{2yz}$

(a) $\nabla f(x,y,z)=\left\langle f_x(x,y,z),f_y(x,y,z),f_z(x,y,z)\right\rangle=\left\langle e^{2yz},2xze^{2yz},2xye^{2yz}\right\rangle$

(b) $\nabla f(3,0,2)=\langle 1,12,0\rangle$

(c) By Equation 14, $D_{\mathbf{u}} f(3,0,2)=\nabla f(3,0,2)\cdot \mathbf{u}=\langle 1,12,0\rangle\cdot \left\langle \frac{2}{3},-\frac{2}{3},\frac{1}{3}\right\rangle=\frac{2}{3}-\frac{24}{3}+0=-\frac{22}{3}$.

15. $f(x,y,z)=\sqrt{x^2+y^2+z^2}\Rightarrow \nabla f(x,y,z)=\left\langle \frac{x}{\sqrt{x^2+y^2+z^2}},\frac{y}{\sqrt{x^2+y^2+z^2}},\frac{z}{\sqrt{x^2+y^2+z^2}}\right\rangle$,

$\nabla f(1,2,-2)=\left\langle \frac{1}{3},\frac{2}{3},-\frac{2}{3}\right\rangle$, and a unit vector in the direction of **v** is

$\mathbf{u}=\frac{1}{9}\langle -6,6,-3\rangle=\left\langle -\frac{2}{3},\frac{2}{3},-\frac{1}{3}\right\rangle$, so

$D_{\mathbf{u}} f(1,2,-2)=\nabla f(1,2,-2)\cdot \mathbf{u}=\left\langle \frac{1}{3},\frac{2}{3},-\frac{2}{3}\right\rangle\cdot\left\langle -\frac{2}{3},\frac{2}{3},-\frac{1}{3}\right\rangle=\frac{4}{9}$.

23. $f(x,y)=\sin(xy)\Rightarrow \nabla f(x,y)=\langle y\cos(xy),x\cos(xy)\rangle$, $\nabla f(1,0)=\langle 0,1\rangle$. Thus the maximum rate of change is $|\nabla f(1,0)|=1$ in the direction $\langle 0,1\rangle$.

27. **(a)** As in the proof of Theorem 15, $D_{\mathbf{u}} f=|\nabla f|\cos\theta$. Since the minimum value of $\cos\theta$ is -1 occurring when $\theta=\pi$, the minimum value of $D_{\mathbf{u}} f$ is $-|\nabla f|$ occurring when $\theta=\pi$, that is when **u** is in the opposite direction of ∇f (assuming $\nabla f\neq\mathbf{0}$).

(b) $f(x,y)=x^4y-x^2y^3\Rightarrow \nabla f(x,y)=\left\langle 4x^3y-2xy^3,x^4-3x^2y^2\right\rangle$, so f decreases fastest at the point (2,–3) in the direction $-\nabla f(2,-3)=-\langle 12,-92\rangle=\langle -12,92\rangle$.

32. $\nabla T = -400e^{-x^2-3y^2-9z^2}\langle x,3y,9z\rangle$

(a) $\mathbf{u} = \frac{1}{\sqrt{6}}\langle 1,-2,1\rangle$, $\nabla T(2,-1,2) = -400e^{-43}\langle 2,-3,18\rangle$ and

$D_{\mathbf{u}}T(2,-1,2) = \left(-\frac{400e^{-43}}{\sqrt{6}}\right)(26) = -\frac{5200\sqrt{6}}{3e^{43}}$ C°/m.

(b) $\nabla T(2,-1,2) = 400e^{-43}\langle -2,3,-18\rangle$ or equivalently $\langle -2,3,-18\rangle$.

(c) $|\nabla T| = 400e^{-x^2-3y^2-9z^2}\sqrt{x^2+9y^2+8z^2}$ $^\circ$ C/m is the maximum rate of increase. At $(2,-1,2)$ the maximum rate of increase is $400e^{-43}\sqrt{337}$ $^\circ$ C/m.