7. $f(x,y)=x^2/\left(x^2+y^2\right)$. First approach (0,0) along the x –axis. Then $f(x,0)=x^2/x^2=1$ for $x\neq 0$, so $f(x,y)\rightarrow 1$. Now approach (0,0) along the y –axis. Then for $y\neq 0$, $f(0,y)=0$, so $f(x,y)\rightarrow 0$. Since f has two different limits along two different lines, the limit does not exist.

10. $f(x,y)=6x^3y/\left(2x^4+y^4\right)$. On the x –axis, $f(x,0)=0$ for $x\neq 0$, so $f(x,y)\rightarrow 0$ as $(x,y)\rightarrow(0,0)$ along the x –axis. Approaching $(0,0)$ along the line $y=x$ gives $f(x,x)=6x^4/\left(3x^4\right)=2$ for $x\neq 0$, so along this line $f(x,y)\rightarrow 2$ as $(x,y)\rightarrow(0,0)$. Thus the limit does not exist.

11. $f(x,y)=\dfrac{xy}{\sqrt{x^2+y^2}}$. We can see that the limit along any line through $(0,0)$ is 0 , as well as along other paths through $(0,0)$ such as $x=y^2$ and $y=x^2$. So we suspect that the limit exists and equals 0 ; we use the Squeeze Theorem to prove our assertion. $0\leq\left|\dfrac{xy}{\sqrt{x^2+y^2}}\right|\leq|x|$ since $|y|\leq\sqrt{x^2+y^2}$, and $|x|\rightarrow 0$ as $(x,y)\rightarrow(0,0)$. So $\lim\limits_{(x,y)\rightarrow(0,0)} f(x,y)=0$.

28. $F(x,y)=\dfrac{x-y}{1+x^2+y^2}$ is a rational function and thus is continuous on its domain R^2 (since the denominator is never zero).

37. $\lim\limits_{(x,y)\rightarrow(0,0)}\dfrac{x^3+y^3}{x^2+y^2}=\lim\limits_{r\rightarrow 0^+}\dfrac{(r\cos\theta)^3+(r\sin\theta)^3}{r^2}=\lim\limits_{r\rightarrow 0^+}\left(r\cos^3\theta+r\sin^3\theta\right)=0$

39.

$$\lim_{(x,y,z)\rightarrow(0,0,0)}\frac{xyz}{x^2+y^2+z^2}=\lim_{\rho\rightarrow 0^+}\frac{(\rho\sin\phi\cos\theta)(\rho\sin\phi\sin\theta)(\rho\cos\phi)}{\rho^2}$$

$$=\lim_{\rho\rightarrow 0^+}\left(\rho\sin^2\phi\cos\phi\sin\theta\cos\theta\right)=0$$