4. $\mathbf{r}'(t)=\langle 2t,2,1/t\rangle$, $|\mathbf{r}'(t)|=\sqrt{4t^2+4+(1/t)^2}=\dfrac{1+2t^2}{|t|}=\dfrac{1+2t^2}{t}$ for $1\le t\le e$.

$$L=\int_1^e \frac{1+2t^2}{t}\,dt=\int_1^e\left(\frac{1}{t}+2t\right)dt=\Big[\ln t+t^2\Big]_1^e=e^2$$

10. $\mathbf{r}'(t)=2e^{2t}(\cos 2t-\sin 2t)\,\mathbf{i}+2e^{2t}(\cos 2t+\sin 2t)\,\mathbf{k}$,

$$\frac{ds}{dt}=|\mathbf{r}'(t)|=2e^{2t}\sqrt{(\cos 2t-\sin 2t)^2+(\cos 2t+\sin 2t)^2}=2e^{2t}\sqrt{2\cos^2 2t+2\sin^2 2t}=2\sqrt{2}\,e^{2t}\,.$$

$$s=s(t)=\int_0^t |\mathbf{r}'(u)|\,du=\int_0^t 2\sqrt{2}\,e^{2u}\,du=\Big[\sqrt{2}\,e^{2u}\Big]_0^t=\sqrt{2}\,(e^{2t}-1)\Rightarrow$$

$\dfrac{s}{\sqrt{2}}+1=e^{2t}\Rightarrow t=\dfrac{1}{2}\ln\left(\dfrac{s}{\sqrt{2}}+1\right)$. Substituting, we have

$$\begin{aligned}\mathbf{r}(t(s)) &= e^{2\left(\frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)}\cos 2\left(\frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)\mathbf{i}+2\,\mathbf{j}\\ &\quad+e^{2\left(\frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)}\sin 2\left(\frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)\mathbf{k}\\ &=\left(\frac{s}{\sqrt{2}}+1\right)\cos\left(\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)\mathbf{i}+2\,\mathbf{j}+\left(\frac{s}{\sqrt{2}}+1\right)\sin\left(\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)\mathbf{k}\,.\end{aligned}$$

16. **(a)** $\mathbf{T}(t)=\dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}=\dfrac{1}{\sqrt{4t^2+4+(1/t)^2}}\langle 2t,2,1/t\rangle=\dfrac{|t|}{2t^2+1}\langle 2t,2,1/t\rangle$. But since the $\mathbf{k}$ –component is $\ln t$, t is positive, $|t|=t$ and

$\mathbf{T}(t)=\dfrac{1}{2t^2+1}\left\langle 2t^2,2t,1\right\rangle$. Then

$\mathbf{T}'(t)=\dfrac{1}{2t^2+1}\langle 4t,2,0\rangle-\left(2t^2+1\right)^{-2}(4t)\left\langle 2t^2,2t,1\right\rangle=\dfrac{1}{(2t^2+1)^2}\left\langle 4t,2-4t^2,-4t\right\rangle$, so

$$\mathbf{N}(t)=\frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}=\frac{\left\langle 4t,2-4t^2,-4t\right\rangle}{\sqrt{(4t)^2+(2-4t^2)^2+(-4t)^2}}=\frac{1}{2t^2+1}\left\langle 2t,1-2t^2,-2t\right\rangle\,.$$

(b) $\kappa(t)=\dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}=\dfrac{2}{2t^2+1}\left(\dfrac{t}{2t^2+1}\right)=\dfrac{2t}{(2t^2+1)^2}$

18. $\boldsymbol{r}'(t)=\boldsymbol{i}+\boldsymbol{j}+2t\,\boldsymbol{k}$, $\boldsymbol{r}''(t)=2\,\boldsymbol{k}$, $\left|\boldsymbol{r}'(t)\right|=\sqrt{1^2+1^2+(2t)^2}=\sqrt{4t^2+2}$, $\boldsymbol{r}'(t)\times\boldsymbol{r}''(t)=2\,\boldsymbol{i}-2\,\boldsymbol{j}$,
$\left|\boldsymbol{r}'(t)\times\boldsymbol{r}''(t)\right|=\sqrt{2^2+2^2+0^2}=\sqrt{8}=2\sqrt{2}$. Then

$$\kappa(t)=\frac{\left|\boldsymbol{r}'(t)\times\boldsymbol{r}''(t)\right|}{\left|\boldsymbol{r}'(t)\right|^3}=\frac{2\sqrt{2}}{\left(\sqrt{4t^2+2}\right)^3}=\frac{2\sqrt{2}}{\left(\sqrt{2}\,\sqrt{2t^2+1}\right)^3}=\frac{1}{(2t^2+1)^{3/2}}\,.$$

24. $f(x)=\cos x$, $f'(x)=-\sin x$, $f''(x)=-\cos x$,

$$\kappa(x)=\frac{\left|f''(x)\right|}{\left[1+(f'(x))^2\right]^{3/2}}=\frac{\left|-\cos x\right|}{\left[1+(-\sin x)^2\right]^{3/2}}=\frac{\left|\cos x\right|}{(1+\sin^2 x)^{3/2}}$$

40. $(1,0,1)$ corresponds to $t=0$. $\boldsymbol{r}(t)=e^t\left\langle 1,\sin t,\cos t\right\rangle$, so
$\boldsymbol{r}'(t)=e^t\left\langle 1,\sin t,\cos t\right\rangle+e^t\left\langle 0,\cos t,-\sin t\right\rangle=e^t\left\langle 1,\sin t+\cos t,\cos t-\sin t\right\rangle$ and

$$\boldsymbol{T}(t)=\frac{\boldsymbol{r}'(t)}{\left|\boldsymbol{r}'(t)\right|}=\frac{e^t\left\langle 1,\sin t+\cos t,\cos t-\sin t\right\rangle}{e^t\sqrt{1+\sin^2 t+2\sin t\cos t+\cos^2 t+\cos^2 t-2\sin t\cos t+\sin^2 t}}$$
$$=\frac{\left\langle 1,\sin t+\cos t,\cos t-\sin t\right\rangle}{\sqrt{3}}\ ,$$

$\boldsymbol{T}(0)=\left\langle \frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right\rangle$. $\boldsymbol{T}'(t)=\frac{1}{\sqrt{3}}\left\langle 0,\cos t-\sin t,-\sin t-\cos t\right\rangle$, so

$$\boldsymbol{N}(t)=\frac{\boldsymbol{T}'(t)}{\left|\boldsymbol{T}'(t)\right|}=\frac{\frac{1}{\sqrt{3}}\left\langle 0,\cos t-\sin t,-\sin t-\cos t\right\rangle}{\frac{1}{\sqrt{3}}\sqrt{0^2+\cos^2 t-2\cos t\sin t+\sin^2 t+\sin^2 t+2\sin t\cos t+\cos^2 t}}$$
$$=\frac{1}{\sqrt{2}}\left\langle 0,\cos t-\sin t,-\sin t-\cos t\right\rangle\ .$$

$\boldsymbol{N}(0)=\left\langle 0,\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right\rangle$ and $\boldsymbol{B}(0)=\boldsymbol{T}(0)\times\boldsymbol{N}(0)=\left\langle -\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}\right\rangle$.