

10.  $r^2 = x^2 + y^2 = 3^2 + 3^2 = 18$  so  $r = \sqrt{18} = 3\sqrt{2}$ ;  $\tan \theta = \frac{y}{x} = \frac{3}{3} = 1$  and the point  $(3,3)$  is in the first quadrant of the  $xy$ -plane, so  $\theta = \frac{\pi}{4} + 2n\pi$ ;  $z = -2$ . Thus, one set of cylindrical coordinates is  $\left(3\sqrt{2}, \frac{\pi}{4}, -2\right)$ .

20.  $\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{0+3+1} = 2$ ,  $\cos \phi = \frac{z}{\rho} = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$ , and  $\cos \theta = \frac{x}{\rho \sin \phi} = \frac{0}{2 \sin(\pi/3)} = 0 \Rightarrow \theta = \frac{\pi}{2}$  (since  $y > 0$ ). Thus spherical coordinates are  $\left(2, \frac{\pi}{2}, \frac{\pi}{3}\right)$ .

25.  $\rho = \sqrt{r^2 + z^2} = \sqrt{3+1} = 2$ ;  $\theta = \frac{\pi}{2}$ ;  $\cos \phi = \frac{z}{\rho} = \frac{-1}{2} \Rightarrow \phi = \frac{2\pi}{3}$ , so in spherical coordinates the point is  $\left(2, \frac{\pi}{2}, \frac{2\pi}{3}\right)$ .

26.  $\rho = \sqrt{16+9} = 5$ ;  $\theta = \frac{\pi}{8}$ ;  $\cos \phi = \frac{3}{5} \Rightarrow \phi = \cos^{-1}\left(\frac{3}{5}\right)$ , so in spherical coordinates the point is  $\left(5, \frac{\pi}{8}, \cos^{-1}\left(\frac{3}{5}\right)\right) \approx \left(5, \frac{\pi}{8}, 0.927\right)$ .

30.  $z = 4 \cos \frac{\pi}{3} = 2$ ,  $r = 4 \sin \frac{\pi}{3} = 2\sqrt{3}$ ,  $\theta = \frac{\pi}{4}$  and the point is  $\left(2\sqrt{3}, \frac{\pi}{4}, 2\right)$ .

56. (a)  $z = r^2(\cos^2 \theta - \sin^2 \theta)$  or  $z = r^2 \cos 2\theta$ .

(b)  $\rho \cos \phi = \rho^2 \sin^2 \phi (\cos^2 \theta - \sin^2 \theta)$  or  $\cos \phi = \rho \sin^2 \phi \cos 2\theta$ .