

2.

$$\begin{aligned} \mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 1 & 4 \\ -1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 5 & 4 \\ -1 & 2 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 5 & 1 \\ -1 & 0 \end{vmatrix} \mathbf{k} \\ &= (2-0)\mathbf{i} - [10-(-4)]\mathbf{j} + [0-(-1)]\mathbf{k} = 2\mathbf{i} - 14\mathbf{j} + \mathbf{k} \end{aligned}$$

Now  $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = \langle 2, -14, 1 \rangle \cdot \langle 5, 1, 4 \rangle = 10 - 14 + 4 = 0$  and  $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = \langle 2, -14, 1 \rangle \cdot \langle -1, 0, 2 \rangle = -2 + 0 + 2 = 0$ , so  $\mathbf{a} \times \mathbf{b}$  is orthogonal to both  $\mathbf{a}$  and  $\mathbf{b}$ .

6.

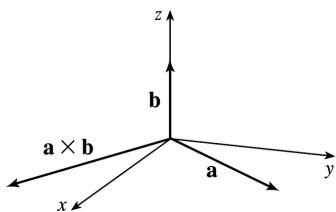
$$\begin{aligned} \mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & e^t & e^{-t} \\ 2 & e^t & -e^{-t} \end{vmatrix} = \begin{vmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & e^{-t} \\ 2 & -e^{-t} \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & e^t \\ 2 & e^t \end{vmatrix} \mathbf{k} \\ &= (-1-1)\mathbf{i} - (-e^{-t} - 2e^{-t})\mathbf{j} + (e^t - 2e^t)\mathbf{k} = -2\mathbf{i} + 3e^{-t}\mathbf{j} - e^t\mathbf{k} \end{aligned}$$

Since  $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = (-2\mathbf{i} + 3e^{-t}\mathbf{j} - e^t\mathbf{k}) \cdot (\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}) = -2 + 3 - 1 = 0$ ,  $\mathbf{a} \times \mathbf{b}$  is orthogonal to  $\mathbf{a}$ .

Since  $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = (-2\mathbf{i} + 3e^{-t}\mathbf{j} - e^t\mathbf{k}) \cdot (2\mathbf{i} + e^t\mathbf{j} - e^{-t}\mathbf{k}) = -4 + 3 + 1 = 0$ ,  $\mathbf{a} \times \mathbf{b}$  is orthogonal to  $\mathbf{b}$ .

12. (a)  $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin \theta = 3 \cdot 2 \cdot \sin \frac{\pi}{2} = 6$

(b)  $\mathbf{a} \times \mathbf{b}$  is orthogonal to  $\mathbf{k}$ , so it lies in the  $xy$ -plane, and its  $z$ -coordinate is 0. By the right-hand rule, its  $y$ -component is negative and its  $x$ -component is positive.



26. (a)  $\vec{PQ} = \langle -3, 2, -1 \rangle$  and  $\vec{PR} = \langle 1, -1, 1 \rangle$ , so a vector orthogonal to the plane through  $P$ ,  $Q$ , and  $R$  is  $\vec{PQ} \times \vec{PR} = \langle (2)(1) - (-1)(-1), (-1)(1) - (-3)(1), (-3)(-1) - (2)(1) \rangle = \langle 1, 2, 1 \rangle$  (or any scalar multiple thereof).

(b) The area of the parallelogram determined by  $\vec{PQ}$  and  $\vec{PR}$  is

$$|\vec{PQ} \times \vec{PR}| = |\langle 1, 2, 1 \rangle| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}, \text{ so the area of triangle } PQR \text{ is } \frac{1}{2} \sqrt{6}.$$

30.

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 1 \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = -2 - 2 + 0 = -4.$$

So the volume of the parallelepiped determined by  $\mathbf{a}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$  is  $|-4| = 4$  cubic units.

32.  $\mathbf{a} = \vec{PQ} = \langle 2, 3, 3 \rangle$ ,  $\mathbf{b} = \vec{PR} = \langle -1, -1, -1 \rangle$  and  $\mathbf{c} = \vec{PS} = \langle 6, -2, 2 \rangle$ .

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} 2 & 3 & 3 \\ -1 & -1 & -1 \\ 6 & -2 & 2 \end{vmatrix} = 2 \begin{vmatrix} -1 & -1 \\ -2 & 2 \end{vmatrix} - 3 \begin{vmatrix} -1 & -1 \\ 6 & 2 \end{vmatrix} + 3 \begin{vmatrix} -1 & -1 \\ 6 & -2 \end{vmatrix} = -8 - 12 + 24 = 4,$$

so the volume of the parallelepiped is 4 cubic units.