Seminar: Diophantine Geometry (Fall 2023)

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1 Logistics

- When: Thursday, 4:20-5:50 PM ET
- Where: TBD
- Organizer: Xiaorun Wu

2 Syllabus

a Module 1 (3-4 Weeks): Weil Height, Roth's theorem

1. Heights in projective & affine space
2. Heights of polynomials
3. Local Heights
4. Global Heights
5. Weil Heights & explicit bonds
6. Metrized line bundles and local heights
7. Siegel's Lemma
8. Proof of Roth's Theorem

b Module 2 (2-3 Weeks): Abelian varieties, Néron-Tate height

1. Group varieties, elliptic curve, picard variety
2. Theorem of square and dual abelian variety
3. Theroem of cube
4. Isogeny multiplication by n
5. Néron-Tate height
6. Néron Symbol
7. Hilbert's irreducibility theorem

c Module 3 (2 Weeks): Mordell-Weil Theorem

1. Weak Mordell-Weil for elliptic curve
2. Weak Mordell-Weil for abelian varieties
3. Hilbert's irreducibility theorem

d Module 4 (3 Weeks): Falting's Theorem, abc-conjecture

1. Vojta divisor
2. Mumford's method for upper bound
3. Vojta's divisor of small height
4. Proof of Falting's theorem
5. Belyi's theorem
6. abc-conjecture

e Module 5 (2 Weeks): Nevanlinna theory, Vojta's conjectures

1. Nevanlinna theory
2. Ahlfors-Shimizu characteristic
3. Holomorphic curves in Nevanlinna theorem
4. Vojta's dictionary & conjectures
5. abc-theorem for function fields