EXTRA PROBLEMS FOR HOMEWORK 10

Problem 11: Suppose that $f(z)$ is analytic, and $\mathrm{Im}(f(z))$ is constant. Show that $f(z)$ is constant. (Note that this implies that $f(z) = \mathrm{Re}(z) = x$ is *not* analytic.)

Problem 12: Consider the function $u(x, y) = \sin x \cosh y$. (Recall that the hyperbolic cosine function is $\cosh y = (e^y + e^{-y})/2$.) Show that $u(x, y)$ is harmonic. Find (using the method of partial integration) a harmonic conjugate $v(x, y)$. Can you recognize the complex analytic function $f(z) = u(x, y) + iv(x, y)$ as something simple? If so, what?

Extra Credit: Suppose that $f(z)$ is analytic, and $\arg f(z)$ is constant. Show that $f(z)$ is constant.