

1 AQFT, its original aim and what was actually achieved up to now

QFT as QM was born in form of a parallelism to classical physics named quantization. In due time (as the result of works of von Neumann, Weyl, Mackey..., see also recent papers by Klaas Landsman) it became quite clear that QM admits an autonomous formulation and the main role of classical setting is to “baptize” interactions by a nice classical names. The non-autonomous Lagrangian quantization setting (in which form Pascual Jordan discovered QFT) was more persevering within QFT; although as far back as 1929 he pleaded for an autonomous QFT, in his words a QFT “without classical crutches”, the first concrete attempts in this direction were only undertaken after the discovery of renormalized perturbation theory. The first step was taken by Irvine Segal. His C^* algebra setting lacked a pivotal physical property: causal localizability. Thanks to Haag’s profound conceptual grip on QFT, the setting of AQFT was to a large extent in place at the beginnings of the 60s. Its (verbal) description was as follows : autonomous QFT is a spacetime indexed net of operator algebras which may have (or not have) field generators. Pointlike field generators are necessarily singular objects (“operator-valued distributions”).

The analogy to modern coordinate-free differential geometry is obvious. There is a large class of pointlike fields (Borchers class) which act in the same Hilbert space and (if smeared with test functions having their support in the same given spacetime region) generate the aforementioned spacetime-indexed algebra. However in contrast to coordinates in differential geometry (which are usually chosen to be regular), the field coordinatization is inexorably singular (well-defined as operator-valued distributions but delicate to handle in multiplicative operations), the regular “invariant” objects are really the spacetime-localized operator algebras. Under certain technical assumption one can pass from the net of spacetime-indexed operator algebras to the generating field-coordinatizations and backward. The case of chiral nets is the most favored since one can pass from the net formulation to the pointlike field generators without additional technical assumptions. Please be aware that the connection with differential geometry is only an analogy i.e. the latter does not help at all to achieve autonomy of QFT (this is the point where I think Peter would disagree with me).

The 70s and 80s were years of impressive structural progress in AQFT. One of the most remarkable structural discoveries is that the observable net already determines all the other (charged) representations (the work of Doplicher, Haag and Roberts, see Haag’s book) which together with the vacuum representation make up the full field-algebra i.e. all charge-transfer operators from one to another representations. This reconstruction of the full theory (charges, their statistics...) from its observable shadow is the solution of a conceptually extremely ambitious “inverse problem” resembling Mark Kac’s famous “how to hear the shape of a drum”. You cannot even think of doing this in a Lagrangian quantization setting.

The last decade is characterized by constructive use of AQFT. On the perturbative side there is the already mentioned generalization of renormalized perturbation theory to curved spacetime. But the most unexpected progress is presently happening in nonperturbative AQFT. It turned out that by combining the AQFT setting with the idea of on-shell particle based concepts as the S-matrix, one arrives at a very powerful new constructive setting which I will refer to as the setting of modular localization. The mathematical basis of this new completely intrinsic (no quantization reference to classical theory) approach is the Tomita-Takesaki theory which is part of operator algebra theory. Next to the Hilbert space theory of quantum mechanics modular theory is the second great meeting ground between physics and mathematics. Different from the Hilbert space theory and the role of differential geometry in physics, modular theory was not a piece of ready made mathematics which physicists found useful, rather different aspects of modular theory were simultaneously developed by mathematicians and physicists. The physical entrance to this theory was the work of Haag Hugenholz and Winnink on quantum statistical mechanics of open system and their main mathematical contribution was the new conceptual role of the KMS property which in the hands of Kubo, Martin and Schwinger was just a computational trick in order to bypass the computation of Gibbs traces. Later it turned out that modular theory is also related to the most important concepts of QFT: localization and causality. Although this relation was first observed around 1975 by Bisognano and Wichmann it took another two decades to become aware of its constructive power in the form of **modular localization**. To its recent successes belong the final understanding of the localization aspects of the third family of positive energy Wigner representations: the so-called zero mass infinite spin family (more appropriately **helicity tower**). These are representations which have a semiinfinite stringlike localization; they cannot be described in terms of Lagrangian quantization. It seems that this third kind of matter has very unusual thermal properties.

Roughly speaking modular localization is the kind of localization inherent in QFT (where fields are restricted to certain spacetime regions) but modular localization is an intrinsic concept which is completely independent on the kind of field coordinatization one uses for the description of the theory and therefore it fits perfectly with the description in terms of spacetime-indexed local nets of AQFT. It may be seen in analogy to the step from coordinate-based geometry to modern invariant differential geometry. The concepts for intrinsicness of QFT are however more complex and different from those of differential geometry and contrary to a widespread opinion differential geometry contributes very little to a more profound understanding of QFT.

Technically speaking AQFT and on-shell S-matrix concepts match in an interesting way because the S-matrix has in addition to its role in scattering theory an important conceptual place in modular theory. It is contained in the modular objects of wedge algebras in relation to the vacuum state $(\mathcal{A}(W), \Omega)$. For those of you who know that modular theory associates two modular objects with such a situation: a one-parametric modular unitary operator group Δ^{it} and a reflection J which is closely related to the famous antiunitary TCP re-

flection. The S-matrix accounts for the difference between the interacting and the free J . As the Lagrangian approach starts from the form of the classical Lagrangian, the approach built on modular localization starts from the algebraic structure of generators of wedge algebras $\mathcal{A}(W)$. There is as yet no general classification theory for such generators but there is an important subclass which permits such a classification and which is quite suitable for explaining the general idea behind the modular localization-based approach. This is the special case of vacuum-**polarization-free-generators (PFG)**. These are fields which like free fields when applied to the vacuum generate one-particle states without any additional vacuum polarization admixture. They are however not pointlike localized fields rather their localization region is a wedge. PFGs with good mathematical properties only exist in $d=1+1$ spacetime dimensions and their S-matrix is necessarily purely elastic. It can be shown that tempered PFGs lead precisely to the so called factorizing theories of the two-dimensional bootstrap-formfactor program and the on-shell Fourier transform are precisely those operators fulfilling Zamolodchikov-Faddeev algebra commutation relations. In this sense the modular localization approach explains the recipes of the bootstrap-formfactor construction in terms of the principles of QFT and in particular reveals the spacetime localization properties behind the Z-F algebra operators. The construction of the localized algebras in the net is achieved by forming intersections of wedge algebras and the nontriviality of a particular model is identical to the non-triviality of these intersections. This nourishes the idea that there may exist a truly intrinsic perturbative approach even for general interacting QFT which aims at the perturbative construction of generators of wedge algebras and avoids the computational use of singular pointlike fields; such an approach may finally arrive at a truly intrinsic frontier between admissible and nonsensical QFTs, an insight which the present power counting division between renormalizable/nonrenormalizable cannot reveal.

The conceptual beauty and perfection of AQFT is its interpretive autonomy/intrinsicness. Let me explain this important point in an example. It is common practice to deal with the thermal aspects of QFT by starting with a temperature Gibbs state on a system in a quantization box of volume V and then pass to the thermodynamic limit (with the appropriately normalized correlation functions fulfilling the KMS condition as a relic of the Gibbs property). The intuitive picture which everybody has in mind is that of a sequence of inclusively increasing partial (local) systems which approach from the inside the infinite (open) system to which the laws of thermodynamic equilibrium apply. But this intuitive picture is strictly speaking a **metaphorical image**. All the systems in the sequence are mathematically different (different quantizations) and there is no natural implementation of an inclusive embedding. The interpretive image is that of the acting physicist and not intrinsic aspect of this particular calculation. Fortunately AQFT disposes of a completely intrinsic description of this situation. This description is conceptually more subtle since it is based on the (only known to AQFTists) split property which can be derived with modular localization for theories with a “tempered” degrees of freedom behavior (the nuclearity criterion, see Haag’s book, in more popular terminology: the absence

of a Hagedorn temperature). In this setting the finite system really does become a material part of the open system. Although the detailed fluctuations in this correct treatment are such that for large systems the leading behavior in V is identical to that of the standard metaphorical picture (so that energy and entropy will be proportional to V in both descriptions), the nonleading terms are expected to deviate. Thanks to special properties of holographic images of actual QFTs one can directly map the heat bath thermal behavior on the globalized horizon into the localization thermality in the restricted vacuum state (hep-th/0511291) and obtain e.g. the entropical area law. The crucial property from which all these wonderful results derive are the vacuum fluctuations of the living space (in the sense of modular localization) of quantum matter.

The use of metaphorical arguments in QFT is only a trick to have simpler calculations. In each case of their use in QFT they can be replaced by intrinsic (generally more demanding) arguments. Not so in string theory. As a result of the fact that quantum matter in a string theory description receives its only quantum fluctuation aspect from the 2-dimensional source space (i.e. the chiral field theory) and not from the target space, all statements (already from its very beginning) are metaphorical. You can open any paper on string theory, take for example Verlinde's power point images of the recent 2006 string meeting, all the spacetime interpretations of modular forms (to the extent that they go beyond Gibbs temperatures) are metaphorical. This makes string theory extraordinarily eery and unreal from a physical viewpoint, never mind its mathematical sophistication. The cause of this is the absence of direct quantum fluctuations in target space. This is what I mean by "mismatch" of scale-sliding arguments string \rightarrow QFT and structural arguments. The credibility of the former require the validity of the latter.

In case there are further questions on this pivotal point I will come back to it in the form of direct answers.