Quantum Field Theory for Mathematicians: Background and History

This course is intended as an introduction to quantum field theory for mathematicians, although physicists may also find some of the material here to be of interest. Much of the course will be devoted to working out basic examples of quantum field theories, especially those that have been of mathematical interest.

Quantum field theory is not a subject which is at the point that it can be developed axiomatically on a rigorous basis. There are various sets of axioms that have been proposed (for instance Wightman's axioms for non-gauge theories on Minkowski space or Segal's axioms for conformal field theory), but each of these only captures a limited class of examples. Many quantum field theories that are of great interest have so far resisted any useful rigorous formulation. This course will try and operate at a somewhat higher degree of rigor than that of the standard arguments used by physicists, at least to the extent of making clear what parts of the subject do have a precise rigorous treatment, and which don't. In general, the emphasis will not be on careful analysis, but instead on explaining the interesting mathematical structures that have appeared in physicist's calculations.

1 Suggested Reading

One motivation for producing these notes is that there are few good places for mathematicians to learn the basics of quantum field theory. On of the best is the notes [4] of the 1996-7 special year on quantum field theory held at the Institute for Advanced Study. The emphasis of these notes will be much more on the basics and simple examples than the IAS notes. Perhaps these notes will help anyone who wants to tackle the IAS notes. Another useful volume is the proceedings of the IAS/Park City 1991 Summer School [6], especially the lecture of Jeff Rabin and Orlando Alvarez.

Some of the standard textbooks used to teach quantum field theory courses to physicists are, in historical order [16], [8], [1], [9], [12], [2] and [3], [7], [11], [10]. Of these [9] and [11] have the virtue of conciseness, with [11] having a much more modern viewpoint.

Some other books that may be helpful are [14], which covers similar material to the standard textbooks, but with somewhat more mathematical care, [5] which just covers quantum gauge theories, a new chatty introductory book by Zee [17], and an exhaustive three-volume treatment by Weinberg [15].

2 Some History

It is useful to have some knowledge of the history of a subject when beginning to study it, so what follows is a short outline of the history of quantum field theory, including in particular the history of those parts of the theory most relevant to mathematicians. For a detailed history that covers some of this, see [13]

- 1925: Matrix mechanics version of quantum mechanics (Heisenberg)
- 1925-26: Wave mechanics version of quantum mechanics, Schrödinger equation (Schrödinger)
- 1927-29: Quantum field theory of electrodynamics (Dirac, Heisenberg, Jordan, Pauli)
- 1928: Dirac equation (Dirac)
- 1929: Gauge symmetry of electrodynamics (London, Weyl)
- 1931: Heisenberg algebra and group (Weyl), Stone-von Neumann theorem
- 1948: Feynman path integral forumlation of quantum mechanics
- 1948-49: Renormalized quantum electrodynamics (QED) (Feynman, Tomonoga, Schwinger, Dyson)
- 1954: Non-abelian gauge symmetry, Yang-Mills action (Yang, Mills, Shaw, Utiyama)
- 1959: Wightman axioms (Wightman)
- 1962-3: Segal-Shale-Weil representation (Segal, Shale, Weil)
- 1967: Glashow-Weinberg-Salam gauge theory of weak interactions (Weinberg, Salam)
- 1971: Renormalized non-abelian gauge theory ('t Hooft)
- 1971-72: Supersymmetry
- 1973: Non-abelian gauge theory of strong interactions (QCD) (Gross, Wilczek, Politzer)
- 1974: Representations of Kac-Moody algebras (Kac)
- 1975: Instanton solutions to Yang-Mills equations (Belavin, Polyakov, Schwarz, Tyupkin)
- 1978: Vertex operators (Frenkel, Lepowsky, others)
- 1982: Donaldson invariants of four-manifolds (Donaldson)
- 1983: Conformal field theory (Belavin, Polyakov, Zamolodchikov)
- 1983: Wess-Zumino-Witten model (Witten)
- 1988: Topological quantum field theories of Donaldson invariants, knot polynomials and Gromov-Witten invariants (Witten)

- 1990-1: Mirror symmetry (Greene, Plesser, Candelas)
- 1994: Seiberg-Witten equations (Seiberg, Witten)

References

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[10] Peskin, M. and Schroeder, *An Introduction to Quantum Field Theory,* Westview Press, 1995.

[11] Ramond, P., *Field Theory: A Modern Primer,* Benjamin-Cummings, 1981.

[12] Schweber, S., *An Introduction to Relativistic Quantum Field Theory,* Harper and Row, 1961.

[13] Schweber, S., *QED and the Men Who Made It,* Princeton University Press, 1994.

[14] Ticciati, R., *Quantum Field Theory for Mathematicians,* Cambridge University Press, 1999.

[15] Weinberg, S., *The Quantum Theory of Fields (Three volumes),* Cambridge University Press, 1995-2000.

[16] Wentzel, G., *Quantum Theory of Fields,* Interscience Publishers, 1949.

[17] Zee, A., *Quantum Field Theory in a Nutshell* Princeton University Press, 2003.