

# The Challenge of Unifying Particle Physics

Peter Woit

Columbia University

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# Outline

## 1 Introduction

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- 2 Quantum Mechanics and Representations of Groups

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- Happy to be more controversial in discussions later.

# Quantum Mechanics: 1926 (Heisenberg, Schrodinger, Dirac)

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## Observables

Possible “observable” physical quantities are functions on this Phase Space.

# Quantum Mechanics: States

In quantum mechanics the state of the world at a time  $t$  is described completely by a vector

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in a “Hilbert space”  $\mathcal{H}$ . A Hilbert space is just a space of vectors with a notion of length

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# Quantum Mechanics: Observables

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In quantum mechanics, observable physical quantities correspond not to numbers but to “self-adjoint operators  $A$ ” on Hilbert space. These are analogs in infinite dimensions of matrices, with “self-adjointness” meaning that the eigenvalues are real.

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## Interpretation of Quantum Mechanics

The eigenvalues of  $A$  give the possible numerical values of the physical quantity corresponding to  $A$ . Some states (“eigenvectors”) have well defined eigenvalues, others don't. In the latter case, one ends up only able to predict the probability of observing different values.

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- Phase (of a complex number) symmetry: Charge  $Q$

# Representation Theory: 1925 (Weyl)

At almost the same time that quantum mechanics was being developed, Schrodinger's colleague and friend in Zurich, the mathematician Hermann Weyl, was developing the theory of “representations of groups” This has become a central part of modern mathematics. It is also very closely related to quantum mechanics.

# Representation Theory: 1925 (Weyl)

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Hard to explain in this kind of lecture, but I'll try, by giving some examples.

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- Rotations in the complex plane:  $G = U(1)$

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Whenever a physical system has a group of symmetries  $G$ , its quantum mechanical state space  $\mathcal{H}$  is a representation of  $G$ . These symmetries leave invariant the length  $\Psi(t)$  of state vectors, so mathematicians call this kind of representation “unitary”.

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Let’s see how our examples work.

# Time Translation Symmetry, $G = \mathbf{R}$

If the environment acting on a physical system is not changing with time, the physical system has a symmetry under translations in time. The group of these translations  $t \rightarrow t + \Delta t$  is  $G = \mathbf{R}$ , the real line. The Hilbert space  $\mathcal{H}$  is a representation of this symmetry, with the group acting on the state vectors in it by

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$H$  is the “Hamiltonian operator”, one of our observables. It is a linear operator (like a matrix) that acts on state vectors. Its eigenvalues give the possible values of the energy.

# Space Translation Symmetry, $G = \mathbf{R}^3$

If the environment acting on a physical system is the same no matter where one is in space, the physical system has a symmetry under translations in the three spatial directions. The group of these translations  $x_i \rightarrow x_i + \Delta x_i$  is  $G = \mathbf{R}^3$ , and the Hilbert space  $\mathcal{H}$  is a representation of  $G$ .

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If we represent states as “wave-functions”, i.e. functions  $\Psi(x_1, x_2, x_3, t)$  depending on the location in space, then we have

$$\frac{d}{dx_i} \Psi(x_1, x_2, x_3, t) = \sqrt{-1} P_i \Psi(x_1, x_2, x_3, t)$$

The eigenvalues of  $P_i$  are the possible  $i$ 'th components of the momentum.

# Rotations in Space, $G = O(3)$

If the environment acting on a physical system only depends on the distance from a central point (same in all directions), the physical system has a symmetry under rotations about that point. The group of these rotations is tricky to visualize and deal with, but it is three-dimensional and mathematicians call it  $O(3)$ . Again, the Hilbert space is a representation of the group.

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The operators analogous to  $P_i$  that implement infinitesimal rotations about the  $x_i$  axis are conventionally called  $J_i$ . These operators are trickier to deal with, because it matters in which order we apply them. A lot of a typical quantum mechanics course is devoted to showing how these work, essentially to working out the representation theory of  $O(3)$ . The bottom line is that states are labelled by a number called “spin”, with possible values

$$0, \frac{1}{2}, 1, \frac{3}{2}, \dots$$

# Rotations in the Complex Plane, $G = U(1)$

Quantum mechanics has a symmetry not seen in classical mechanics since it is based on complex numbers. We can rotate our complex numbers in the complex plane by an angle  $\theta$  often getting a new symmetry of the theory. Mathematicians call this group of rotations in the complex plane  $U(1)$ , and the Hilbert space  $\mathcal{H}$  is a representation of it.

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The operator analogous to  $H$  that implements infinitesimal rotations of this kind is called  $Q$ . It operates on  $\mathcal{H}$ , and its eigenvalues have to be integers (since, as you rotate around the complex plane once, the action on  $\mathcal{H}$  has to come back to where it started, but can rotate around any integer number of times).



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These integer eigenvalues correspond to the charge of the state, can be

$$\dots, -2, -1, 0, 1, 2, \dots$$

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- The mathematical structure of quantum mechanics is closely connected to what mathematicians call representation theory. This is a central, unifying theme in mathematics.
- The Hilbert space of quantum mechanical states  $\mathcal{H}$  of a system is a representation of the groups of symmetries of the system.
- Much of the physics of the system is determined by this, including the behavior of four of the most important observables (energy, momentum, spin and charge), which correspond to four different symmetries (time translation, space translation, spatial rotations, complex plane rotations).

# Quantum Field Theory

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By 1929, this theory had been written down, and was called Quantum Electrodynamics (QED).

# QED Symmetries: Translations

In QED, space and time are treated together according to Einstein's special relativity.

## Translations

Separate space ( $\mathbf{R}^3$ ) and time ( $\mathbf{R}$ ) translations are combined into space-time ( $\mathbf{R}^4$ ) translations. Energy and momentum operators combine into an energy-momentum operator.

# QED Symmetries: Lorentz Group

## Lorentz Group

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## Poincare Group

Combining the Lorentz group and space-time translations gives a group called the “Poincare group”.

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The two kinds of fields in QED have a different nature with respect to gauge symmetry:

- The electron and proton fields take complex values that are transformed independently at different places
- The electromagnetic field is what mathematicians call a connection: it tells one how to compare the values of fields at neighboring points, even after doing independent transformations on them.

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- The group of gauge symmetries is infinite dimensional. To this day, the representation theory of this group is not understood. Physicists generally believe this doesn’t matter, that only the “trivial” representation matters, not the rest. In other words, one just needs to understand the “gauge-invariant” part of the theory.

# General Gauge Theories (Yang, Mills) 1954

In 1954 Yang and Mills constructed a generalization of QED, by replacing the role of the group  $U(1)$  by a larger group, called  $SU(2)$ , generalizing the notion of gauge symmetry. Quantum field theories of this kind are now called “Gauge Theories”.

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- Renormalization: Veltman, 't Hooft (1971)

# The Standard Model

By the 1950s many kinds of particles besides the electron and proton had been discovered, and, besides the electromagnetic force, there were two others: the weak force responsible for nuclear beta-decay, and the strong force responsible for binding quarks together into protons and neutrons, and thus into atomic nuclei.

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## The Glashow-Weinberg-Salam Model, 1967

This is a gauge theory with group  $SU(2) \times U(1)$  which extends QED to explain both the electromagnetic and weak forces. The crucial insight needed to make it work was that one needed to include a mechanism to make the vacuum state not invariant under the  $SU(2)$  symmetry. This is now called the “Higgs mechanism” and uses a different kind of field, which predicts the existence of a “Higgs particle” that has not yet been observed.

# The Standard Model: QCD

Before 1973, conventional wisdom was that the strong force could not be described by a quantum field theory. Much research was devoted to alternatives to quantum field theory, including early versions of “string theory”, a theory based not on fields, but on replacing particles with very different 1-dimensional elementary objects. At long distances the forces between quarks were very strong, but got weak at short distances, something not supposed to happen in a quantum field theory.

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Before 1973, conventional wisdom was that the strong force could not be described by a quantum field theory. Much research was devoted to alternatives to quantum field theory, including early versions of “string theory”, a theory based not on fields, but on replacing particles with very different 1-dimensional elementary objects. At long distances the forces between quarks were very strong, but got weak at short distances, something not supposed to happen in a quantum field theory.

## Quantum Chromodynamics

In 1973, using the new renormalization techniques Gross, Wilczek and Politzer showed that some gauge theories were “asymptotically free”: the force disappeared at short distance and particles moved freely, while it became strong at large distances. So the strong force could be described by a gauge theory and there was exactly one viable possibility: the gauge theory with group  $SU(3)$ , in which quarks came in 3 kinds, called “colors”. This was a generalization of QED, so baptized QCD

# The Standard Model: Successes

The combination of the Glashow-Weinberg-Salam model and QCD is a gauge theory with group  $SU(3) \times SU(2) \times U(1)$ . It was from the beginning very successful at explaining known experimental results and predicting new ones, so quickly got the name “Standard Model”.



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To a large extent, the victim of its own success.

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- Why do particles have the masses that they have?
- What about gravity? The quantum field theory of the gravitational force is not renormalizable by known methods.

# Speculative Ideas: Grand Unified Theories

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Doesn't explain Higgs mechanism, introduces new Higgs Mechanism to explain why we don't see full  $SU(5)$ .

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In simplest version, expect particles to occur in pairs (“superpartners”), not what we see in the real world.



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- 1997 AdS/CFT: Revival of strings for strong interactions

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- Mid 90s: More complicated structures that strings can begin and end on: branes

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Lesson of mid-90s: Branes lead to all sorts of new things you can do with strings, but also lead to many more possibilities, many more different kinds of possible unified physics.

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No testable predictions about physics at all so far from this picture.

Debate rages as to whether any are possible.

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Debate rages over whether this is science at all. No proposals for predictions.

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This explanation is not scientifically testable, very dangerous for people who want to argue with Intelligent Designers to abandon standard notions of scientific testability.

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Maybe future progress will require not just unification of physics, but unification with mathematics...