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RESEARCH ARTICLE

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Topological defect solutions in the spherically symmetric space-time admitting conformal motion

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Abstract In this paper, we have examined strings with monopole and electric field and domain walls with matter and electric field in the spherically symmetric space-time admitting a one-parameter group of conformal motions. For this purpose, we have solved Einstein's field equations for a spherically symmetric space-time via conformal motions. Also, we have discussed the features of the obtained solutions.

Keywords Domain wall · String · Monopole · Conformal motion

1 Introduction

It is still a challenging problem to know the exact physical situation at very early stages of the formation of our universe. At the very early stages of evolution of universe, it is generally assumed that during the phase transition (as the universe passes through its critical temperature) the symmetry of the universe is broken spontaneously.

Spontaneous symmetry breaking is an old idea, described within the particle physics context in terms of the Higgs field. The symmetry is called spontaneously broken if the ground state is not invariant under the full symmetry of the lagrangian density. Thus, the vacuum expectation value of the Higgs field is nonzero. In quantum field theories, broken symmetries are restored at high enough temperatures.

Spontaneous symmetry breaking can give rise to topologically stable defects. Topological defects [1, 2] are stable field configurations that arise in field theories with spontaneously broken discrete or continuous symmetries. Depending on the topology of the vacuum manifold M they are usually identified as domain walls

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[2] when $M = Z_2$, as strings [3] and one-dimensional textures [4, 5] when $M = S^1$, as monopoles [6–8] and two dimensional textures when $M = S^2$ and three dimensional textures when $M = S^3$. Depending on whether the symmetry is local (gauged) or global (rigid), topological defects are called local or global They are expected to be remnants of phase transitions that may have occurred in the early universe. They also form in various condensed matter systems which undergo low temperature transitions [9].

In the case in which the phase transition is induced by the Higgs sector of the Standard Model, the defects are domain walls across which the field flips from one minimum to the other. The defect density is then related to the domain size and the dynamics of the domain walls is governed by the surface tension σ.

It is clear that a full analysis of the role of domain walls in the Universe imposes the study of their interaction with particles in the primordial plasma.

The presence of zero modes localized on domain wall can be important for the stability of the wall. In particular, fermionic zero modes may give rise to interesting phenomena as the magnetization of domain walls [10] and the dynamical generation of massive ferromagnetic domain walls [11]. Indeed, fermionic zero modes could drastically change both gravitational properties and cosmic evolution of a gas of domain walls.

The interaction of scalar particles and Dirac fermions with a domain wall has been the object of various papers in the literature (see [1] and references there in).

Of all these cosmological structures, strings have excited the most interest. The present day configurations of the universe are not contradicted by the large scale network of strings in the early universe. Moreover, they may act as gravitational lenses and may give rise to density fluctuations leading to the formations of galaxies [12].

In String Theory, the myriad of particle types is replaced by a single fundamental building block, a 'string'. These strings can be closed, like loops, or open, like a hair. As the string moves through time it traces out a tube or a sheet, according to whether it is closed or open, Furthermore, the string is free to vibrate, and different vibrational modes of the string represent the different particle types, since different modes are seen as different masses or spins.

Monopoles are point like topological objects that may arise during phase transitions in the early universe.

In this study, we will attach monopole to the strings and normal matter to the domain walls. It is plausible to attach these objects. Because higher dimensional objects may contain the lower dimensional objects. We will solve Einstein's field equations for spherical symmetric space-times via Conformal Killing Vector (CKV).

General Relativity provides a rich arena to use symmetries in order to understand the natural relation between geometry and matter furnished by the Einstein equations. Symmetries of geometrical/physical relevant quantities of this theory are known as collineations. The most useful collineations are conformal Killing vectors.

Conformal collineation is defined by,

$$\pounds_\xi g_{ab} = 2\psi g_{ab}, \quad \psi = \psi(x^a) \tag{1}$$

where $\pounds_\xi$ signifies the Lie derivative along ξ^a and $\psi(x^a)$ is the conformal factor. In particular, ξ is a special conformal Killing vector (SCKV) if $\psi_{;ab} = 0$ and $\psi_{,a} \neq 0$. Other subcases are homothetic vector (HV) if $\psi_{,a} = 0$ and $\psi \neq 0$, and Killing vector (KV) if $\psi = 0$. Here (;) and (,) denote the covariant and ordinary derivatives, respectively.

Conformal Killing Vectors provide a deeper insight into the space-time geometry and facilitate generation of exact solutions to the field equations.

The study of Conformal Motions in space-time is physically very important. Because, it can lead to the discovery of conservation laws and devise space-time classification schemes. For example, collineations can be considered as non-Noetherian symmetries and can be associated with constants of motion and, up to the level of Conformal Killing Vectors, they can be used to simplify the metric [13]. Affine collineations are related to conserved quantities [14] (a result used to integrate the geodesics in FRW space-times), Ricci collineations are related to the conservation of particle number in FRW space-times [15] and the existence of curvature collineations imply conservation laws for null electro-magnetic fields [16].

Also, the existence of a conformal Killing vector is closely related to "global" equilibrium properties of perfect fluids. For instance, the conformal symmetry singles out a perfect fluid with the equation of state for radiation.

In other words, for massless particles the condition for global equilibrium requires the quantity u^a/T, where u^a is the fluid 4-velocity and T is the fluid temperature, to be a conformal Killing vector (CKV). Only the conformal symmetry of an "optical" metric, in which an effective refraction index of the cosmic substratum characterizes specific internal interactions that macroscopically correspond to a negative pressure contribution, may be compatible with the production of entropy [17–19].

Further more, we are able to reduce partial differential equations to ordinary differential equations by using Conformal motions.

So, in this paper it is imposed the condition that the space time manifold admits a conformal motion. Because the use of conformal motions, instead of homothetic motions($\phi = constant$), allow us to find static and spherically symmetric distributions of matter which may be fitted to the exterior Schwarzschild metric [20]. Also, we have chosen to study spherically symmetric space-times since, other than FRW space-times, the majority of the remaining Conformal Motion space-times known to the authors are spherically symmetric.

Conformal collineations have been studied at length by various authors. Herrera et al. [21] have studied conformal collineations, with particular reference to perfect fluids and anisotropic fluids; Duggal and Sharma [22] extend this work to the more general case of a special affine collineation ξ^a; Coley and Tupper [23] have discussed spacetimes admitting Special Conformal Killing Vector and symmetry inheritance; Mason and Maartens [24] have considered kinematics and dynamics of conformal collineations; Maartens et al. [25] have studied the conformal collineations in anisotropic fluids, in which they are particularly concerned with Special Conformal Killing Vectors. Coley and Tupper [26] and Maartens [27] have studied conformal motion in the spherically symmetric space-times. Carot et al. [28] have discussed space-times with conformal motions. Recently, work on symmetries of the string has been taken by Yavuz and Yilmaz [29]. Also Yilmaz [30] has studied timelike and spacelike Ricci collineation vectors in the string.

The paper is outlined as follows. In Sect. 2, Einstein field equations and their solutions are obtained for monopole attached to the strings in the Spherically symmetric space-times by using conformal motions . In the Sect. 3, solutions of the Einstein field equations are obtained for normal matter attached to the domain wall via conformal motions depending on conformal factor i.e., $\psi(x^a)$. In Sect. 4, concluding remarks are given.

2 Einstein's Field equations and their solutions for the strings with monopole and electric field

Let us consider a static distribution of matter represented by charged spherical symmetric matter which may be monopole attached to the string.

In Schwarzschild coordinates the line element takes the following form:

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\Omega^2 \tag{2}$$

with

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2, \qquad x^{1,2,3,4} \equiv r, \theta, \phi, t$$

The total energy-momentum tensor T_{ab} is assumed to be the sum of three parts, T^S_{ab}, T^M_{ab} and T^E_{ab}, for string, monopole and electromagnetic contributions, respectively, i.e.,

$$T_{ab} = T^S_{ab} + T^M_{ab} + T^E_{ab}. \tag{3}$$

The energy-momentum tensor for string [31] is given by

$$T^S_{ab} = \rho_s(U_a U_b - X_a X_b) \tag{4}$$

where ρ_s is string tension density.

Also, here U^a is the four velocity $U^a = \delta^a_4 e^{-\nu/2}$, X^a is the unit spacelike vector in the radial direction $X^a = \delta^a_1 e^{-\lambda/2}$ which represent the strings directions, i.e. the direction of anisotropy. The energy momentum tensor for monopole [1] is given by

$$T^M_{ab} = \partial_a\phi^i\partial_b\phi^i - g_{ab}L \tag{5}$$

where Lagrangian, L, scalar potential, $V(\phi)$ and scalar field, ϕ^i are given as follows, respectively,

$$L = \frac{1}{2}\partial_a\phi^i\partial^a\phi^i - V(\phi) \tag{6}$$

$$V(\phi) = \frac{1}{4}\lambda(\phi^i\phi^i - \eta^2)^2 \tag{7}$$

and

$$\phi^i = \eta h(r)\frac{x^i}{r}. \tag{8}$$

$$T^E_{ab} = -\frac{1}{4\pi}\left(F^c_a F_{bc} - \frac{1}{4}g_{ab}F_{ef}F^{ef}\right) \tag{9}$$

where F_{ab} is the electromagnetic field tensor defined in terms of the four-potential A_a as

$$F_{ab} = A_{b;a} - A_{a;b}. \tag{10}$$

For the electromagnetic field we shall adopt the gauge

$$A_a(0, 0, 0, \phi(r)). \tag{11}$$

Einstein-Maxwell equations can be expressed as

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}, \tag{12}$$

$$F_{ab;c} + F_{bc;a} + F_{ca;b} = 0, \tag{13}$$

$$F^{ab}_{;b} = -4\pi J^a, \tag{14}$$

where J^a is the four-current density that becomes $J^a = \bar{\rho}_e U^a$ and $\bar{\rho}_e$ is the proper charge density. Here we shall use geometrized units so that $8\pi G = c = 1$. Using the line element Eq. (2), the field equations Eqs. (3)–(13) take the form,

$$\rho_s + \frac{\eta^2}{r^2} + E^2 = -e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda\prime}{r}\right) + \frac{1}{r^2}, \tag{15}$$

$$\rho_s + \frac{\eta^2}{r^2} + E^2 = -e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu\prime}{r}\right) + \frac{1}{r^2}, \tag{16}$$

$$E^2 = \frac{e^{-\lambda}}{2}\left(\nu\prime\prime + \frac{\nu\prime 2}{2} + \frac{\nu\prime - \lambda\prime}{r} - \frac{\nu\prime\lambda\prime}{2}\right), \tag{17}$$

$$[r^2 E(r)]' = 4\pi \rho_e r^2. \tag{18}$$

Primes denote differentiation with respect to r, and E is the usual electric field intensity defined as

$$\begin{aligned} F_{41}F^{41} &= -E^2, \\ E(r) &= -e^{-(\nu+\lambda)/2}\phi\prime(r), \\ \phi\prime(r) &= F_{14} = -F_{41}. \end{aligned} \tag{19}$$

The charge density ρ_e defined in Eq. (18) is related to the proper charge density $\overline{\rho}_e$ by

$$\rho_e = \overline{\rho}_e e^{\lambda/2}. \tag{20}$$

Now we shall assume that space-time admits a one-parameter group of conformal motions Eq. (1), i.e.,

$$\pounds_\xi g_{ab} = \xi_{a;b} + \xi_{b;a} = \psi g_{ab}, \tag{21}$$

where ψ is an arbitrary functions of r. From Eqs. (2) and (21) and by virtue of spherical symmetry, we get the following expressions

$$\xi^1 \nu' = \psi, \tag{22}$$

$$\xi^4 = C_1 = const, \tag{23}$$

$$\xi^1 = \psi r/2, \tag{24}$$

$$\lambda' \xi^1 + 2\xi^1_{,1} = \psi, \tag{25}$$

where a comma denotes partial derivatives. From Eqs. (22)–(25), we get

$$e^{\nu} = C_2^2 r^2, \tag{26}$$

$$e^{\lambda} = \left(\frac{C_3}{\psi}\right)^2, \tag{27}$$

$$\xi^a = C_1 \delta_4^a + (\psi r/2)\delta_1^a, \tag{28}$$

where C_2 and C_3 are constants of integration [32]. Expressions Eqs. (26)–(28) contain all the implications derived from the existence of the conformal collineation.

Now substituting Eqs. (26) and (27) into Eqs. (15)–(17), we have

$$\rho_s + \frac{\eta^2}{r^2} + E^2 = (1/r^2)(1 - \psi^2/C_3^2) - 2\psi\psi'/C_3^2 r, \tag{29}$$

$$\rho_s + \frac{\eta^2}{r^2} + E^2 = (1/r^2)(1 - 3\psi^2/C_3^2), \tag{30}$$

$$E^2 = \psi^2/C_3^2 r^2 + 2\psi\psi'/C_3^2 r. \tag{31}$$

From Eqs. (29) and (30) we get

$$\psi = C_4 r \tag{32}$$

If we substitute Eq. (32) into Eqs. (29)–(31) we have

$$\rho_s = \frac{1}{r^2}(1 - \eta^2) - \frac{6C_4}{C_3^2}, \tag{33}$$

$$E^2 = \frac{3C_4^2}{C_3^2}. \tag{34}$$

Using Eqs. (26) and (27), the line element Eq. (2) becomes

$$ds^2 = C_2^2 r^2 dt^2 - \frac{C_3^2}{\psi^2} dr^2 - r^2 d\Omega^2, \tag{35}$$

Let us now consider that the charged sphere extends to radius r_0. Then the solution of Einstein-Maxwell equations for $r > r_0$ is given by the Reissner-Nordström metric as

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2, \tag{36}$$

and the radial electric field is

$$E = q/r^2, \tag{37}$$

where M and q are the total mass and charge, respectively.

To match the line element Eq. (35) with the Reissner-Nordström metric across the boundary $r = r_0$ we require continuity of gravitational potential g_{ab} at $r = r_0$

$$(C_2 r_0)^2 = \left(\frac{\psi}{C_3}\right)^2 = 1 - \frac{2M}{r_0} + \frac{q^2}{r_0^2}, \tag{38}$$

and also we require the continuity of the electric field, which leads to

$$E(r_0) = \frac{q}{r_0^2}, \tag{39}$$

From Eq. (37) and left hand side of Eq. (34) we get

$$\frac{q^2}{r_0^2} = 3r_0^2 \frac{C_4^2}{C_3^2}. \tag{40}$$

Feeding this expression and Eq. (32) back into Eq. (38) we obtain

$$\frac{M}{r_0} = \frac{1}{2} + \left(\frac{C_4}{C_3}\right)^2 r_0^2, \tag{41}$$

or from Eqs. (40) and (41) we have

$$M = \frac{r_0}{2} + \frac{1}{3}\frac{q^2}{r_0}, \tag{42}$$

3 Einstein's Field equations and their solutions for the domain Walls with matter and electric field

In this section, we will consider domain walls with matter and electric field in the spherically symmetric-space-times.

The total energy-momentum tensor T_{ab} is assumed to be the sum of two parts, T_{ab}^D and T_{ab}^E, for domain wall and electromagnetic contributions, respectively, i.e.,

$$T_{ab} = T_{ab}^D + T_{ab}^E \tag{43}$$

The energy-momentum tensor of the domain wall [33] is given by

$$T_{ab}^D = (\rho + p)u_a u_b - p g_{ab} \tag{44}$$

where u_a is the four velocity, $u^a = \delta_4^a e^{-\nu/2}$.

Energy-momentum tensor of domain wall includes normal matter described by ρ_m and p_m as well as a domain wall tension σ, i.e. $\rho = \rho_m + \sigma$ and $p = p_m - \sigma$. Also, p_m and ρ_m are related by the equation of state

$$p_m = (\gamma - 1)\rho_m \tag{45}$$

where $1 \le \gamma \le 2$.

Using the line element Eq. (2), from Eqs. (9)–(12) and (44) we get

$$\rho + E^2 = -e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) + \frac{1}{r^2} \tag{46}$$

$$-p + E^2 = -e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) + \frac{1}{r^2} \tag{47}$$

$$p + E^2 = \frac{e^{-\lambda}}{2}\left(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu'\lambda'}{2}\right) \tag{48}$$

$$[r^2 E(r)]' = 4\pi q_e r^2 \tag{49}$$

Primes denote differentiation with respect to r and E is the usual electric field intensity defined as

$$F_{41}F^{41} = -E^2$$
$$E(r) = -e^{-(\nu+\lambda)/2}\phi'(r) \quad (50)$$
$$\phi'(r) = F_{14} = -F_{41}.$$

The change density q_e defined in Eq. (49) is related to the proper charge density $\overline{q_e}$ by

$$q_e = \overline{q_e}e^{\lambda/2}. \quad (51)$$

There are four total field equations with five unknowns, which are ρ, p, E^2, ν and λ. To get solutions of field equations we need one additional assumption.

So, we will solve Einstein field equations by using conformal motions.

Substituting Eqs. (26) and (27) into Eqs. (46)–(48), we have

$$\rho + E^2 = \left(\frac{1}{r^2}\right)\left(1 - \frac{\psi^2}{C_3^2}\right) - \frac{2\psi\psi'}{C_3^2 r} \quad (52)$$

$$-p + E^2 = \left(\frac{1}{r^2}\right)\left(1 - \frac{3\psi^2}{C_3^2}\right) \quad (53)$$

$$p + E^2 = \frac{\psi^2}{C_3^2 r^2} + \frac{2\psi\psi'}{C_3^2 r} \quad (54)$$

from Eqs. (52)–(54) we get

$$\rho = \frac{1}{2r^2} - \frac{3\psi\psi'}{C_3^2 r} \quad (55)$$

$$p = \frac{1}{r^2}\left(\frac{2\psi^2}{C_3^2} - \frac{1}{2}\right) + \frac{\psi\psi'}{C_3^2 r} \quad (56)$$

$$E^2 = \frac{1}{r^2}\left(\frac{1}{2} - \frac{\psi^2}{C_3^2}\right) + \frac{\psi\psi'}{C_3^2 r} \quad (57)$$

Using Eqs. (26) and (27), the line element Eq. (2) becomes

$$ds^2 = C_2^2 r^2 dt^2 - \frac{C_3^2}{\psi^2}dr^2 - r^2 d\Omega^2 \quad (58)$$

If the function $\psi(r)$ is specified a priori, the problem will be fully determined. According to Eqs. (55)–(57), different solutions can be obtained by specifying the choice of $\psi(r)$. In what follows, instead of arbitrarily assuming function $\psi(r)$, we shall find some solutions by using Eqs. (55)–(57) from physical considerations depending on dynamical quantities ($E^2 = 0$, $\rho = 0$, $p = 0$, $\rho_m = p_m = 0$).

Case (i) If $\psi=\pm\frac{1}{2}(2C_3^2+4r^2C_4)^{1/2}$ then from Eqs. (55)–(57) we get

$$\rho=\frac{1}{2r^2}-\frac{3C_4}{C_3^2 r} \tag{59}$$

$$p=\frac{1}{r^2}+\frac{3C_4}{C_3^2} \tag{60}$$

$$E^2=0 \tag{61}$$

where C_4integration constant.

Using the equation of the state Eq. (45) for the matter, from Eqs. (59) and (60) we get

$$\rho_m=\frac{1}{\gamma r^2} \tag{62}$$

$$p_m=\frac{(\gamma-1)}{\gamma}\frac{1}{r^2} \tag{63}$$

$$\sigma=\frac{1}{r^2}\left(\frac{1}{2}-\frac{1}{\gamma}\right)-\frac{3C_4}{C_3^2}. \tag{64}$$

Case (ii) If $\psi^2=C_3^2\frac{\ln r}{3}+C_5$, from Eqs. (55)–(57) we get the following expressions

$$\rho=0 \tag{65}$$

$$p=\frac{4}{3r^2}\left(\ln r+\frac{3C_5}{C_3^2}-\frac{1}{2}\right) \tag{66}$$

$$E^2=\frac{1}{3r^2}\left(2-\ln r-\frac{3C_5}{C_3^2}\right) \tag{67}$$

where C_5 is integration constant.

Using Eq. (45), from Eqs. (65) and (66) we get

$$\rho_m=\frac{4}{3\gamma r^2}\left(-\frac{1}{2}+\ln r+\frac{3C_5}{C_3^2}\right) \tag{68}$$

$$p_m=\frac{4(\gamma-1)}{3\gamma r^2}\left(-\frac{1}{2}+\ln r+\frac{3C_5}{C_3^2}\right) \tag{69}$$

$$\sigma=-\frac{4}{3\gamma r^2}\left(-\frac{1}{2}+\ln r+\frac{3C_5}{C_3^2}\right) \tag{70}$$

matching the line element Eq. (58) with the Reissner- Nordström metric across the boundary and following the same way in the first section, we obtain the total charge and the total mass as follows

$$\frac{q^2}{r_0^2}=\frac{2}{3}-\frac{\ln r_0}{3}-\frac{C_5}{C_3^2} \tag{71}$$

and

$$M = \frac{r_0}{6} + \frac{q^2}{r_0}. \tag{72}$$

Case (iii) If $\psi^2 = \frac{C_3^2}{4} + \frac{C_6}{r^4}$, then Eqs. (55)–(57) give

$$\rho = \frac{1}{2r^2} + \frac{6C_6}{C_3^2 r^6} \tag{73}$$

$$p = 0 \tag{74}$$

$$E^2 = \frac{1}{2r^2} - \frac{2C_6}{C_3^2 r^6} - \frac{1}{C_3^2 r^2}\left(\frac{C_3^2}{4} + \frac{C_6}{r^4}\right) \tag{75}$$

where C_6 is integration constant.

From Eqs. (45), (73) and (74) we get

$$\rho_m = \frac{1}{\gamma}\left(\frac{1}{2r^2} + \frac{6C_6}{C_3^2 r^6}\right) \tag{76}$$

$$p_m = \sigma = \frac{(\gamma - 1)}{\gamma}\left(\frac{1}{2r^2} + \frac{6C_6}{C_3^2 r^6}\right) \tag{77}$$

In this case, we get the total charge and total mass as follows

$$\frac{q^2}{r_0^2} = \frac{1}{4} - \frac{3C_6}{C_3^2 r_0^4} \tag{78}$$

and

$$M = \frac{r_0}{12} + \frac{2}{3}\frac{q^2}{r_0} \tag{79}$$

Case (iv) If we put $\psi = C_7 r$ and use Eq. (45), from Eqs. (55)–(57) we get

$$\rho = \sigma = \frac{1}{2r^2} - \frac{3C_7^2}{C_3^2} \tag{80}$$

$$p = -\rho = -\sigma = -\frac{1}{2r^2} + \frac{3C_7^2}{C_3^2} \tag{81}$$

$$E^2 = \frac{1}{2r^2} \tag{82}$$

$$\rho_m = 0 \tag{83}$$

$$p_m = 0 \tag{84}$$

where C_7 is integration constant.

In this case, we get the total charge and the total mass as follows

$$\frac{q^2}{r_0^2} = 3r_0^2\left(\frac{C_7}{C_3}\right)^2 \tag{85}$$

and

$$M = \frac{r_0}{2} + \frac{1}{3}\frac{q^2}{r_0}. \tag{86}$$

4 Concluding remarks

In this paper, we have studied charged monopole attached to the string cloud and domain walls with normal matter in the spherical symmetric space-time admitting one-parameter group of conformal motions.

We have obtained the following properties.

a. e^{μ} and e^{λ} are positive, continuous and nonsingular for $r < r_0$.

b. In the case of strings with monopole and electric field, we can conclude that monopole and charge decrease the string's tension. Also, in this case, we have matched our solutions with the Reissner-Nordström metric at $r = r_0$ and obtained black string solutions with monopole. In this situation, we have obtained the increase of the total mass caused by the charge (see Eq. (42)). Also, if $q = 0$ we get total mass for noncharged black string with monopole, i.e., Schwarzschild like black string with monopole. Furthermore, we have obtained uniform charge distribution, i.e., constant electric field.

c. In the case of domain walls with matter and electric field, we have matched our solutions with the Reissner-Nordström metric at $r = r_0$ and obtained black domain wall solutions with matter. In these cases, we have obtained the increase of the total mass caused by the charge (see Eqs. (72), (79) and (86).

In case (i), we have obtained noncharged black domain wall solutions with matter, i.e., Schwarzschild like black domain walls with matter.

In case (ii), we can conclude that while electric field is increasing domain wall's tension and total mass, it decreases the energy density and pressure of the normal matter.

In case (iii), we have obtained that the pressure of the domain walls is equal to the pressure of the matter. In this case, if $\gamma = 1$, then we get only pressureless and charged matter solutions, i.e., domain walls and pressure of the matter disappear.

In case (iv), we have obtained only charged domain wall solutions. In this case, domain walls behave like dark matter due to their negative pressure, i.e., repulsive pressure. We may interpret this solution as charged black hole of charged dark matter.

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