Modern Geometry, Fall 2017: Problem Set 11

Due Thursday, December 7

Problem 1: Show that $(d+\delta)^2$ is minus the conventional Laplacian for the standard metric on $\mathbf{R}^n$.

Problem 2: Show that for $M = \mathbf{R}^4$ with the Minkowski metric, the Maxwell equations in the form

$$dF = 0, \quad d * F = 0$$

are equivalent to the usual vector analysis form of the equations:

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0, \quad \boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 0, \quad \boldsymbol{\nabla} \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t}$$

Problem 3: Show that the Bianchi identity for the curvature Ω on a principal bundle P for the case of $G = SU(2)$ implies that on the base one has the matrix-valued equation

$$dF_\alpha + A_\alpha \wedge F_\alpha - F_\alpha \wedge A_\alpha = 0$$

Problem 4: For the case of Yang-Mills theory ($G = SU(2)$) on $M = \mathbf{R}^4$ with the standard positive-definite metric, write out explicitly the self-duality equation

$$F = *F$$

as a set of partial differential equations for the components of the matrix valued connection form.