# The Angular Correlation of Polarization of Annihilation Radiation

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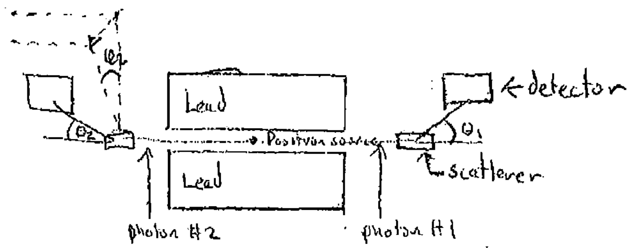
Abstract:

The asymmetry ratio describing the angular correlation of polarization of annihilation radiation was measured, with results, under slightly different conditions, of 2.37 and 2.56; the first value is in accord with theoretical predictions, while the second is slightly higher than expected.

Theory of Angular Correlation of Annihilation Radiation[1]

1. Ideal Geometry




Schematic diagram of the experiment

The Klein-Nishina formula for the Compton scattering cross section is: $\frac{d\sigma}{d\Omega} = \frac{1}{4} r_0^2 \frac{k^2}{k_0^2}\left[\frac{k_0}{k} + \frac{k}{k_0} - 2 + 4\cos^2\Theta\right]$ $k_0$ = initial photon energy, $k$ = scattered photon energy

where: $r_0^2 = \frac{e^4}{16\pi^2 m^2 c^4}$ and $\Theta$ = angle between the initial and final photon polarizations

Energy conservation implies: $k = \frac{k_0 \nu}{\nu + k_0(1-\cos\theta)}$

where: $\nu$ =mass of scattering particle=mass of the electron

In our experiment, since we are using the radiation from positron electron annihilation at rest, $k_0 \simeq \nu$, so $k = \frac{k_0}{2-\cos\theta}$

Notice that the Compton cross section is dependent on the photon polarization. This property of the cross section allows us to use Compton scattering as a polarization analyzer. In this experiment, we will use this property to check the assumption, implied by quantum electrodynamics[2], that the two photons emitted in an electron-positron annihilation are polarized perpendicularly to each other. This will be done by measuring the asymmetry between the case when the two photons are scattered in the same plane, and the case when they are scattered in perpendicular planes. A calculation of this asymmetry follows.

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[1]The discussion in this section is based on: H.S. Snyder, Simon Pasternack, and J. Hornbostel, Phys. Rev. 73, 440-448 (1948)
[2]C.N. Yang, Phys. Rev. 77, 242-245 (1950)

Averaging over the polarization directions of the scattered photon gives: $\frac{\partial\sigma}{\partial\Omega} = \frac{1}{4} r_0^2 \, k^2/k_0^2 \left[ k/k_0 + \frac{k_0}{k} - 2\sin^2\theta\cos^2\phi \right]$

where: $\theta$ =scattering angle

$\phi$ =angle between the scattering plane and the direction of polarization of the incident quantum

So, for an incident photon polarized in the plane of scattering:

$$\left(\frac{\partial\sigma}{\partial\Omega}\right)_{\parallel} = \frac{1}{4} r_0^2 \, k^2/k_0^2 \left[ k/k_0 + \frac{k_0}{k} - 2\sin^2\theta \right]$$

For a photon polarized perpendicularly to the scattering plane:

$$\left(\frac{\partial\sigma}{\partial\Omega}\right)_{\perp} = \frac{1}{4} r_0^2 \frac{k^2}{k_0^2} \left[ \frac{k}{k_0} + \frac{k_0}{k} \right]$$

Since the a <u>priori</u> probability that the incident photon #1 had its polarization in the plane of scattering is $\frac{1}{2}$, and the probability that it had its polarization perpendicular to the scattering plane is also $\frac{1}{2}$, given any photon Compton scattered through an angle $\theta$, with energy $k_1$, the probability that it had its plane of polarization in the plane of scattering is :

$$\frac{(\partial\sigma/\partial\Omega)_{\parallel}}{(\partial\sigma/\partial\Omega)_{\perp} + (\partial\sigma/\partial\Omega)_{\parallel}} = \frac{(k_1/k_0 + k_0/k_1 - 2\sin^2\theta_1)}{2(k_1/k_0 + \frac{k_0}{k_1} - \sin^2\theta_1)}$$

Similarly, the probability that it had its polarization perpendicular to the scattering plane is:

$$\frac{(\partial\sigma/\partial\Omega)_{\perp}}{(\partial\sigma/\partial\Omega)_{\perp} + (\partial\sigma/\partial\Omega)_{\parallel}} = \frac{\left(\frac{k_1}{k_0} + \frac{k_0}{k_1}\right)}{2\left(\frac{k_1}{k_0} + \frac{k_0}{k_1} - \sin^2\theta_1\right)}$$

Since photon #2 is assumed to be polarized perpendicularly to photon #1, the probability that it had its polarization in the scattering plane of photon #1 is: $\left(\frac{k_1}{k_0} + \frac{k_0}{k_1} - 2\sin^2\theta_1\right) / 2\left(\frac{k_1}{k_0} + \frac{k_0}{k_1} - \sin^2\theta_1\right)$

The probability that it had its polarization perpendicular to the scattering plane of photon #1 is: $\left(\frac{k_1}{k_0} + \frac{k_0}{k_1}\right) / 2\left(\frac{k_1}{k_0} + \frac{k_0}{k_1} - \sin^2\theta_1\right)$

Since the Compton cross section for photon #1, averaged over initial and final polarizations is: $\left(\frac{k_1}{k_0} + \frac{k_0}{k_1} - \sin^2\theta_1\right) \frac{1}{4} r_0^2 \frac{k_1^2}{k_0^2}$

the differential cross section for the two photons is given by:

$$\frac{\partial\sigma}{\partial\Omega_1 \partial\Omega_2} = \frac{1}{16} r_0^4 \frac{k_1^2}{k_0^2}\left(\frac{k_1}{k_0} + \frac{k_0}{k_1} - \sin^2\theta_1\right) \left[ \frac{\frac{k_1}{k_0} + k_0/k_1}{2\left(\frac{k_1}{k_0} + \frac{k_0}{k_1} - \sin^2\theta_1\right)} \cdot \frac{k_2^2}{k_0^2}\left(\frac{k_2}{k_0} + \frac{k_0}{k_2} - 2\sin^2\theta_2\cos^2(\phi_2 - \phi_1)\right) \right.$$
$$\left. + \frac{\frac{k_1}{k_0} + \frac{k_0}{k_1} - 2\sin^2\theta_1}{2\left(\frac{k_1}{k_0} + \frac{k_0}{k_1} - \sin^2\theta_1\right)} \cdot \frac{k_2^2}{k_0^2}\left(\frac{k_2}{k_0} + \frac{k_0}{k_2} - 2\sin^2\theta_2\cos^2\left(\phi_2 - \phi_1 - \frac{\pi}{2}\right)\right) \right]$$

Simplifying: $\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{16} r_0^4 \frac{k_1^2 k_2^2}{k_0^4}\left[\gamma_1\gamma_2 - \gamma_1 \sin^2\theta_2 - \gamma_2 \sin^2\theta_1 + 2\sin^2\theta_2 \sin^2\theta_1 \sin^2(\phi_2-\phi_1)\right]$

where: $\gamma_1 = \left(\frac{k_1}{k_0} + \frac{k_0}{k_1}\right)$ and $\gamma_2 = \left(\frac{k_2}{k_0} + \frac{k_0}{k_2}\right)$

$k_1 = \frac{k_0}{2-\cos\theta_1}$ and $k_2 = \frac{k_0}{2-\cos\theta_2}$

For the case of equal scattering angles $\theta = \theta_1 = \theta_2$, $k_1 = k_2 = k$, $\gamma = \gamma_1 = \gamma_2$

$$\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{16} r_0^4 \frac{k^4}{k_0^4}\left[\gamma^2 - 2\gamma\sin^2\theta + 2\sin^4\theta \sin^2(\phi_2-\phi_1)\right]$$

For this case, the ideal asymmetry is given by:

$$\rho = \frac{\frac{d\sigma}{d\Omega_1 d\Omega_2}\Big|_{\phi_2-\phi_1=\pi/2}}{\frac{d\sigma}{d\Omega_1 d\Omega_2}\Big|_{\phi_2-\phi_1=0}} = \frac{\gamma^2 - 2\gamma\sin^2\theta + 2\sin^4\theta}{\gamma^2 - 2\gamma\sin^2\theta} = \boxed{1 + \frac{2\sin^4\theta}{\gamma^2 - 2\gamma\sin^2\theta}}$$

Please remind us here that this is for an angle of 90° between the scattering planes, for comparison with what comes next.

This function has a maximum of 2.85 at $\theta = 82°$.

The asymmetry for an angle of 30° between the scattering planes will be given by:

$$\rho_{30°} = \frac{\frac{d\sigma}{d\Omega_1 d\Omega_2}\Big|_{\phi_2-\phi_1=30°}}{\frac{d\sigma}{d\Omega_1 d\Omega_2}\Big|_{\phi_2-\phi_1=0°}} = 1 + \frac{2\sin^4\theta}{\gamma^2 - 2\gamma\sin^2\theta}\cdot\sin^2(30°)$$

$= 1.46$ for $\theta = 82°$

By a similar calculation: $\rho_{60°} = 2.38$ for $\theta = 82°$, for an angle between the scattering planes of 60°.

The observed asymmetries at 30 and 60 degrees (1.39 and 2.06, see Results section) are somewhat lower than these ideal geometry values, as one would expect. However, these observed values are higher than one would naively compute by taking the finite geometry corrections to be proportional to the asymmetry. On the other hand, we were not able to do the finite geometry corrections for 30° and 60°, so we are unable to make any detailed comparison between theory and experiment.

this seems premature here

2. Effects of a finite geometry

No practical experiment can realize the ideal geometry assumed in the preceding section. The detectors in any experiment will subtend a finite angle, and this must be taken into account. Calculations of the expected asymmetry for detectors subtending various azimuths and scattering angles (assuming that both detectors subtend the same angles) were made in the previously quoted paper by Snyder, Pasternack and Hornbostel. The following is a graph from their paper which summarizes their results:

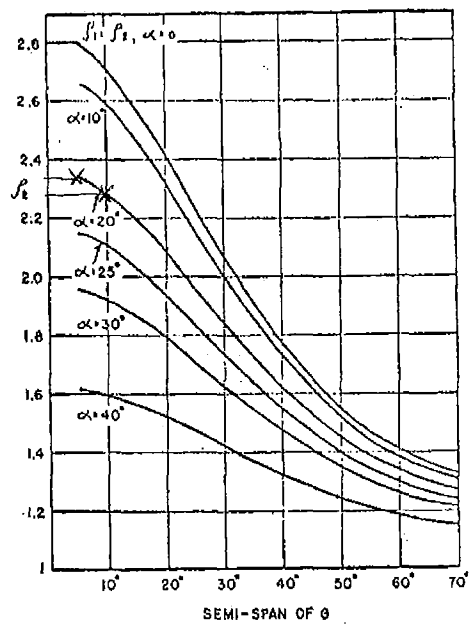



FIG. 4. Asymmetry in coincidence counting rates, $\rho_2 = P_{orthogonal}/P_{coplanar}$, for finite geometry, as function of half-span in $\theta$ for various half-spans in azimuth, $\alpha$. Curve for $\alpha = 0$ also represents asymmetry ratio $\rho_1 = \Delta P_{\phi=\pi/2}/\Delta P_{\phi=0}$.

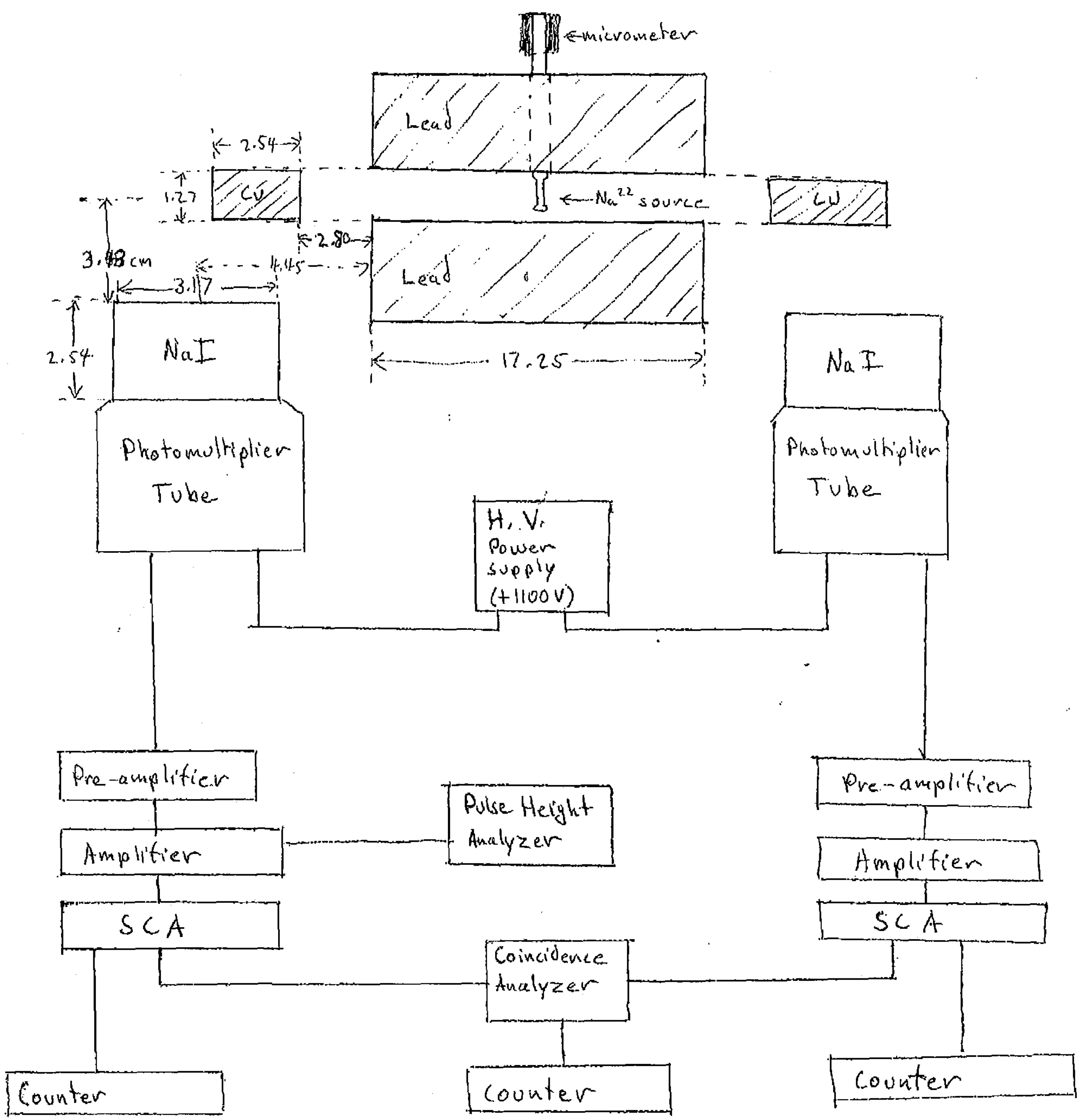
In our experiment $\alpha$ was, to within two degrees, equal to 20°, ~~[illegible]~~ how is $\alpha$ defined?

and the semi-span of θ was approximately 10° for our early runs and 5° for the later ones. For a justification of this statement see the following section which describes the experiment in detail.

Description of the experiment

1. The experiment

See the following figure for a semi-schematic diagram of the experiment:




All dimensions the same on both sides
All dimensions in centimeters.

~~[illegible]~~ decay scheme? figure

<u>The Source</u>: The radioactive source was a sample of $Na^{22}$, which emits a single positron in its disintegration, followed by a 1.2 Mev photon. Its intensity was .45 millicuries (1.66 $X10^7$ disintegrations/sec), which was not checked, but was read off from a scale posted in the laboratory. The emitted positron can either annihilate in flight, or, what is more likely, it can be captured by electrons in the surrounding material, forming positronium, which is unstable and decays into two perpendicularly polarized photons, each with energy of 511 kev, and with opposite momentum. Since the ratio of 511 kev photons to 1.277 Mev photons is 1.78 (from laboratory manual), 100X1.78/2= 89% of the positrons are captured and produce 511 kev photons. In early runs of the experiment, the source was not centered in the lead shielding, in later runs it was (see Results section).

<u>The Scatterer</u>: All of the data was taken using 1" long copper scatterers. These scatterers were chosen since they contained the most electrons and therefore had the highest Compton cross section of the materials available to us. We wished thereby to ensure that as many as possible of the photons were Compton scattered. In retrospect, it seems that the use of 1" aluminum scatterers would have been preferable since, although the rate of observed coincidences would have been lower, the experimental results would not have been as heavily affected by the multiple scattering of the photons in the scatterer.

The mean free path in centimeters for a photon of 511 kev is where is the total scattering cross section and n is the number of electrons per $cm^3$: $n = \frac{N_0 \rho Z}{A} = 2.46X10^{24}/cm^3$

$N_0$: Avogadro's number   $\rho$: density
Z: Atomic Number   A: Atomic weight

$\sigma = 2.8 \times 10^{-25} cm^2$/electron (pg. 88, <u>Graphs of the Compton Energy-Angle Relationship and the Klein-Nishina Formula from 10 kev to 500 Mev</u>) So the mean free path of photons in copper is 1.35 cm. This means that the beam of photons is attenuated as it goes through the scatterer by a factor of $\exp(-2.54/1.35)=.15$. Therefore, 85% of the photons are expected to be scattered. The median point m for photon scattering ( the point for which half of the scattered photons are scattered on each side of it) is determined by the equation : $\int_0^m e^{-x/1.35}dx = \int_m^{2.54} e^{-x/1.35}dx$ and is thus determined to be 1.09 cm, not quite half way through the scatterer. The median scattering angle is determined by this point and using the values from the diagram of the experiment is $90^\circ - \arctan\left(\frac{4.45-(1.09+2.80)}{3.43}\right)$ which turns out to be 81°, very close to the ideal value of 82°.

Could have checked by counting transmission

Note that since the radius of the scatterer is .43 of the mean free path, 100X(1-exp(-.43)))=35% of the photons will be scattered twice before leaving the scatterer.

OK estimate, but strictly speaking they are distributed uniformly across the cross section so that some have to go further than others.

<u>Scintillator</u>: The scintillator consisted of a cylindrical piece of NaI of diameter 1.25", height 1", connected to a photomultiplier tube. 511 kev photons produced negative tail pulses of order .03 volts (see oscilloscope trace pictures following this section)

This 0.03 volts depends on cables + input impedance of scope

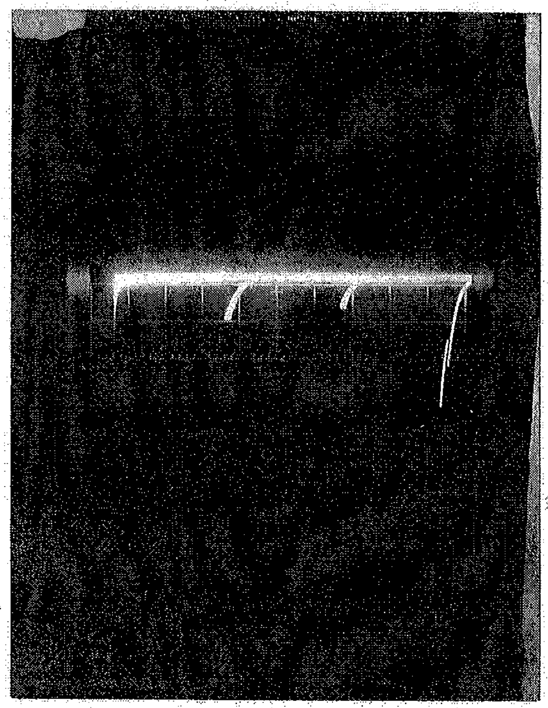
<u>Scintillation Preamplifier</u>: The scintillation preamplifier integrates the charge output signal of the photmultiplier tube, producing a tail pulse? whose height is proportional to the energy deposited by the photon in the scintillator. It also inverts the input pulse, producing a positive output pulse of height around .1v for a .511 Mev photon. (See picture)

Amplifier: The amplifier turns the preamplifier output pulse into a positive pulse of near-Gaussian shape and height around 5v(see pictures).

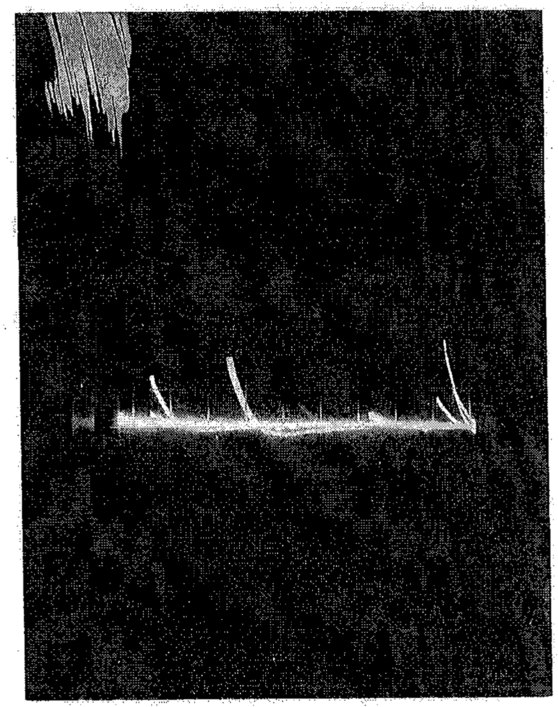
Single Channel Analyzer (SCA): The SCA's were set by using their output to gate the input of the pulse height analyzer, which was used to analyze the pulses coming out of the amplifier. The SCA baseline and window width were adjusted so that only pulses in a narrow band centered about the Compton scattering peak were observed. See the section on the pulse height analysis for the picture of this Compton peak. The Compton peak was rather broad, largely due to the problem of multiple scattering. In early runs of the experiment the window width was rather large, accepting about ½ the Compton peak. Later on, these windows were readjusted, narrowing them to about ¼ the width of the Compton peak. See the pictures for a picture of the 10v, 1 microsecond output logic pulse of the SCA.

Coincidence Analyzer: The early runs of the experiment were done using a resolving time of around 10 nanoseconds. In other words, the coincidence analyzer produced a 10v, 1 microsecond output whenever the leading edges of two pulses, one from each SCA, were within 10 nanoseconds of each other. Later on, since we feared that drifting in the time delay between the two SCA's might be causing problems, we increased this resolving time to .5 microseconds. The delay time between the two SCA's was set by connecting both preamps to the same photomultiplier tube, and varying the delay until the number of coincidences was a maximum, somewhat less than the number of output pulses from the two SCA's since their window settings were not exactly the same.

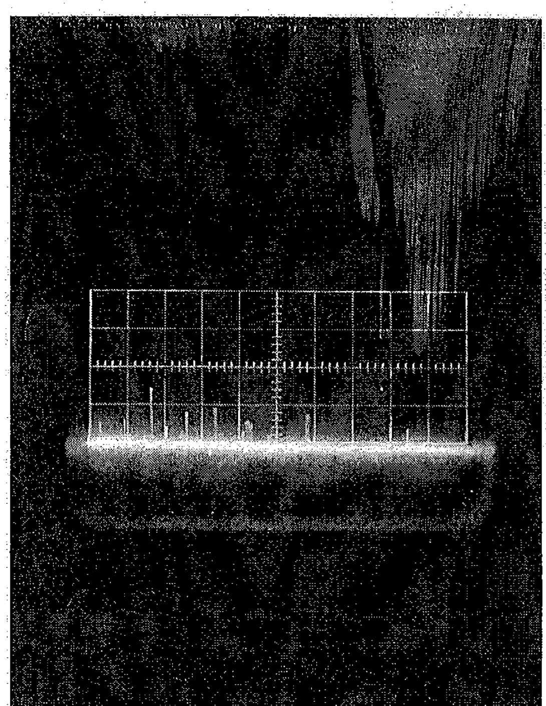
The Counters: Three digital counters were used, one was connected to the output of each SCA, another was connected to the output of the coincidence analyzer.



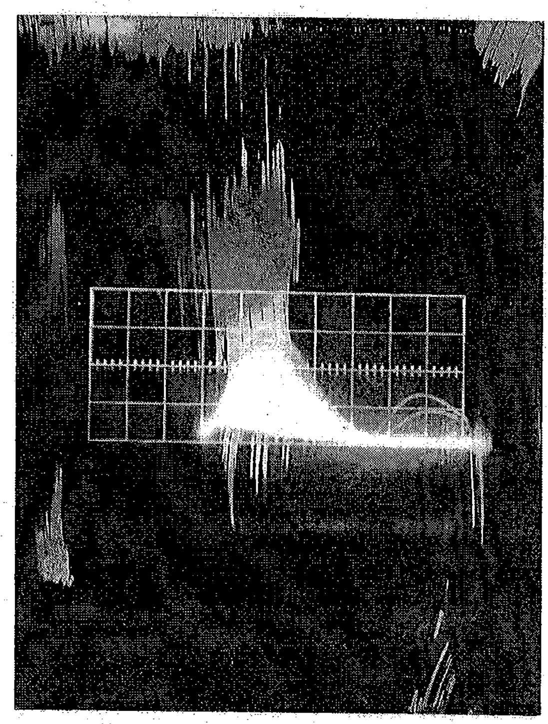
Photomultiplier tube
.05 V/cm .1 msec/cm
1/25 second



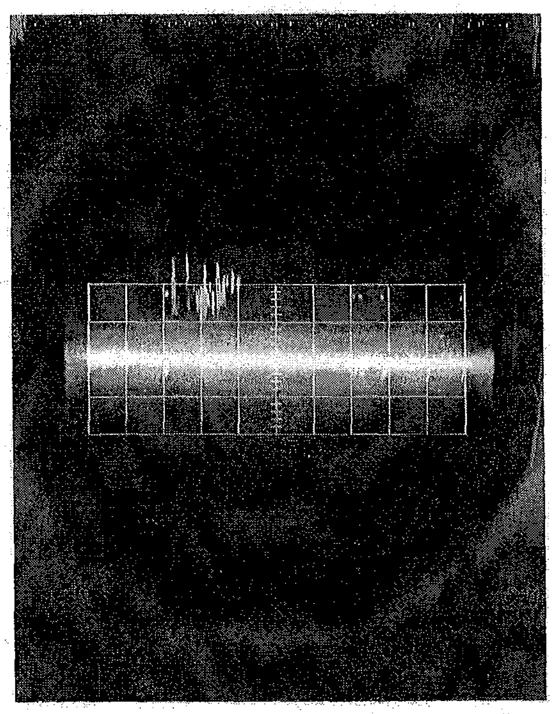
Preamplifier
.1 V/cm .1 msec/cm
1/25 second



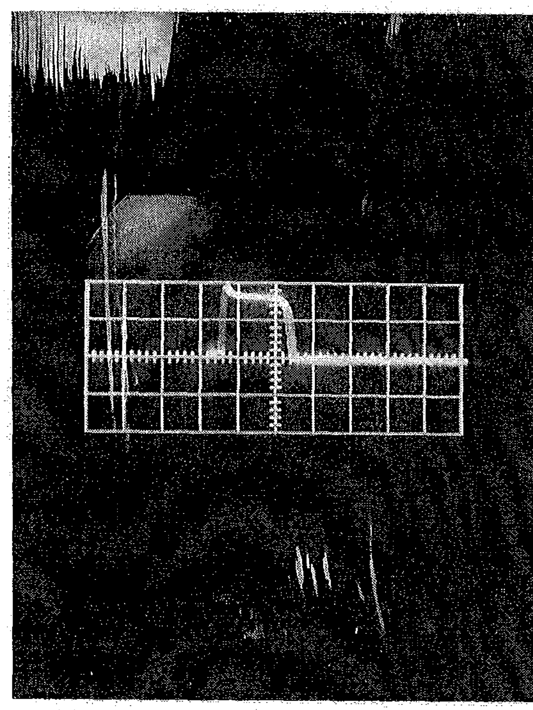
Amplifier
5 V/cm .1 msec/cm
1/10 second



Amplifier
2 V/cm .5 usec/cm
1/10 second



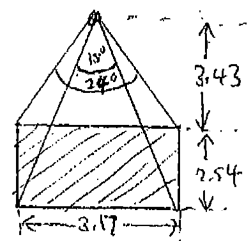
Single Channel Analyzer
2 v/cm .1 msec/cm
1/2 second



Coincidence Analyzer
2 v/cm .5 usec/cm
1/5 second

2. Expected Asymmetry

For our experiment, the scintillator had a diameter of 3.17 cm, and was at a distance of 3.43 cm from the center of the scatterer. If one considers the effective diameter of the scintillator to be that diameter such that any photon that goes through it must have traversed the entire thickness of the scintillator, then the effective angle subtended by the scintillator is to be calculated using the distance from the center of the scatterer to the back of the scintillation material (see diagram):




So, half the effective azimuth subtended by the detector would be $\arctan\left(\frac{3.17/2}{3.43+2.54}\right) = 15^\circ$. However, there will be some contribution from the photons not going through all of the scintillator. Therefore I will use as the effective angle the average of $15^\circ$ and $\arctan\left(\frac{3.17/2}{3.43}\right) = 24^\circ$ (the half angle such that the photons traverse at least some of the detector). As a result, for this experiment, the effective half-azimuth is, to within $\pm 2^\circ$, $20^\circ$. [illegible]

The half span for the scattering angle would seem to be the same, but the use of the SCA's to restrict the energy of the counted photons reduces this by a factor of ½ in the early runs of the experiment and a factor of ¼ in the later runs. This occurs because the only photons counted are those in a narrow energy range, which corresponds to a narrow range of Compton scattering angles. We will thus take $10^\circ$ as the half span in scattering angle for the early runs, and $5^\circ$ for the later runs.

This assumes that the peak width is due totally to the spread of incoming energies due to different scattering angles. But even a monoenergetic source produces a broadened peak in the response. Is the expected variation of energy with angle strong enough to cause the width of the observed peak?

Using the graph reproduced in the section on the theory of the experiment gives: $\rho = 2.30$ for the early runs

$\rho = 2.35$ for the later runs

From examination of this same graph, the approximate $2^{\circ}$ uncertainty in the half-azimuth translates into an approximate uncertainty of .1 in the calculation of the asymmetry parameter.

3. Expected Coincidence Rate

The expected coincidence rate can be roughly determined by taking into account the following factors:

A: # of annihilation photons coming from the source:

.45 millicuriesX.89= $1.48\text{X}10^{7}$ disintegrations/sec

B: Fraction of photons that reach the scatterer:

$(.63)^2\pi/(4\pi(11.42)^2)=.0012$

C: Fraction of photons that are single Compton scattered into the detector= (fraction scattered-fraction double scattered)
X(normalized Compton cross section at 82)*
X(geometrical factor) ← what factor is this?
=(.85-.35)X.4X.051 = .010

where is coincidence involved in this?

D: Efficiency of the detector

From the graph that comes with the scintillator, at 511 kev, the absorption coefficient for the photoelectric effect =.2/cm. Since the photons travel through about 2.5 cm on the average, the efficiency of the scintillator is about 45%.

E: The SCA window is centered on only the center fourth of the peak

The expected rate is given by AXBX$(\text{CXDXE})^2$ = .022/sec
=1.4/min

which is within a factor of 10 of all the observed rates.

how does this "coincidence" rate differ from a calc. of singles rate? Is it squared? Yes maybe the 2nd part is.

*This number is taken from the graph in Heitler, <u>Quantum Theory of Radiation</u>, 3rd ed., pg. 220

4. Expected accidentals rate

With a .5 micro second resolving time and an average singles rate of 200/sec in each channel, we would expect an accidental coincidence rate of $2 \times .5 \times 10^{-6} \times (200)^2 \times 60 = 2.4$ counts/min. This rate occurs because the leading edge of the pulses in each channel open up a window of .5 microsecond, during which a random pulse may come along in the other channel, producing an accidental coincidence. This predicted rate of accidental coincidences is much higher than that which we determined by introducing a delay in one channel during one of the experimental runs(see results). Using that procedure we determined the actual accidentals rate to be about .3 counts/min. The only explanation we can find for this discrepancy is that the calibration of the resolving time dial on the coincidence analyzer is inaccurate.

This is easily tested. It is not in error by factor of 8

Spectrum analysis:

Photo number 1 was taken with the scintillation counter (NaI, Thallium activated) at $0^{o}$ with respect to the beam, with no scatterer; i.e. the direct singles spectrum of the Na 22 source. At the far right is a very prominent peak centered at channel #444, which corresponds to the 511 KeV annihilation photon. We used this known relationship as a calibration. Given that the peak channel number can only be determined to within about 5 channel numbers the values for the energies thus obtained are only good to within around 1%.

Where is the 1.22 MeV?

At 309 KeV there is an edge, and at 199KeV a peak, which are the result of backscattering in the scintillator. To explain this we must look at the Compton scattering formula:

$$\frac{E}{E_0} = \left[1 + \frac{E_0}{m_e c^2}(1 - \cos\theta)\right]^{-1}$$

where E and $E_0$ are the final and initial photon energies respectively. When the photon enters the scintillator it can either get absorbed, giving up all its energy, or it can scatter off something, losing some energy in the process, and escape.

The maximum energy the photon can lose in one scattering event is (for $\theta = 180^{o}$; $E_0 = m_e$) 2/3 $E_0$; by the same reasoning, the least energy deposited in such an event should be 1/3 $E_0$ (For scattering off shield). Actually though, there is a spread in the possible scattering angle, so if we take the mean value of $\cos\theta$ as $-2/\pi$, we get 194 KeV for the peak, so the edge should start at 317 KeV. These are close (within 2.5%) to the observed values.

which channel? The next peak, moving leftwards, is at 89 KeV, which is in excellent agreement with the 87KeV $K\alpha$ x-ray in Lead. This occurs when the incoming photon strikes the Pb shielding in the counter, causing an x-ray to be emitted.

The furthermost left peak, though somewhat smeared out, is probably the $K\alpha$ edge for Iodine. It is located at about 35 KeV which is in good agreement with the 33KeV edge in the absorption plot accompanying the scintillator.

Photo number 2 is the singles spectrum at $90^{o}$ using a Copper scatterer, and number 3 is the same with an Aluminum scatterer. They are essentially the same, except for the great difference in height between the central peak in Cu and that in Al. Also, the right half of the Al spectrum is shifted slightly to the left, due to the fact that the mean free path (for 511 KeV photons) in Al is ~4.3cm which puts the scattering center much closer to being directly over the center of the counter than does the Cu scatterer with a mean free path of ~1.35 cm.

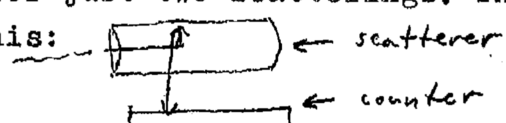
please relate KeV to channel #

Thus the Compton peak ~~in~~ with Al scatterers is at 253 KeV, while that in Cu is at 284 KeV. In both of these cases though, the peaks are broad, ~50 KeV, and there is a long tail corresponding to scattering through the walls of the container. Where did you position the windows?

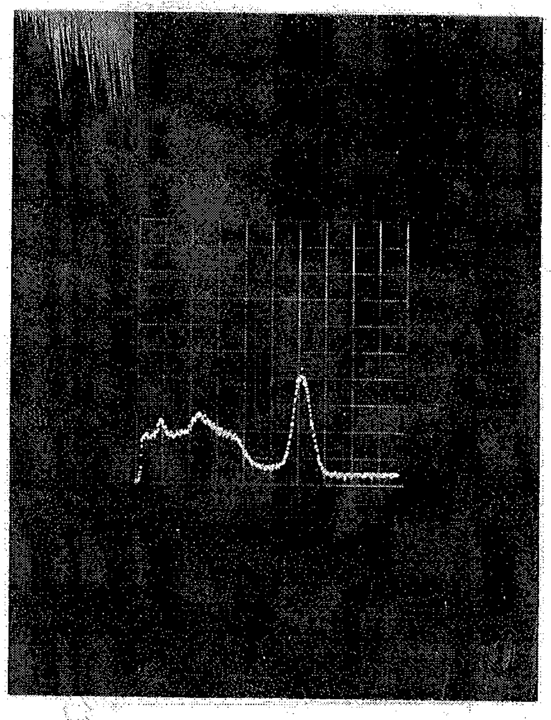
At the left, the Pb and I x-ray peaks are in the same positions as in photo #1, which is to be expected since they are peculiar to the scintillator alone.

The large central peak is due to multiple scattering; i.e. a photon comes into the scatterer and is scattered two or more times within the scatterer before coming into the scintillator. The most likely such event is for just two scatterings. The simplest geometry looks like this:

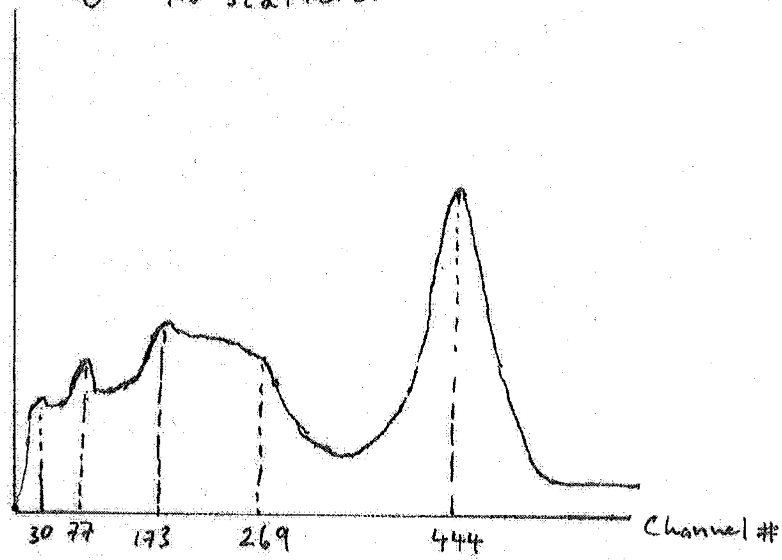
by what standard?



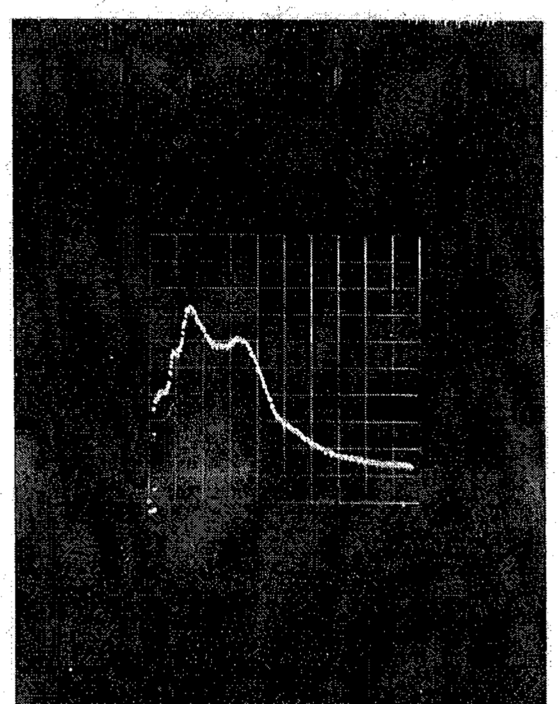

Using the Compton formula twice gives $E = 1/4\ E_0 = 127$ KeV. In Al the observed peak is at 136 KeV, and in Cu at 144 KeV. Considering the scattering center shift described above and the fact that 127 KeV is actually somewhat low for a two-scattering event, these values are in good agreement. This also explains why the peak is so much higher in Cu- the mean free path is much shorter, so there will be more collisions.



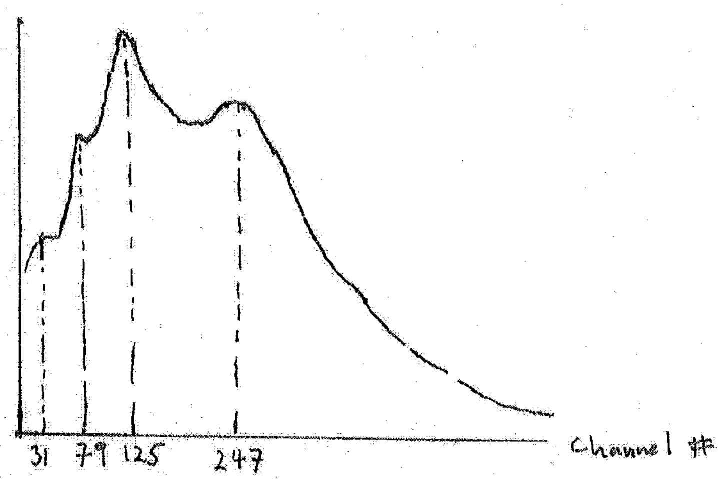
#1
0° No scatterer

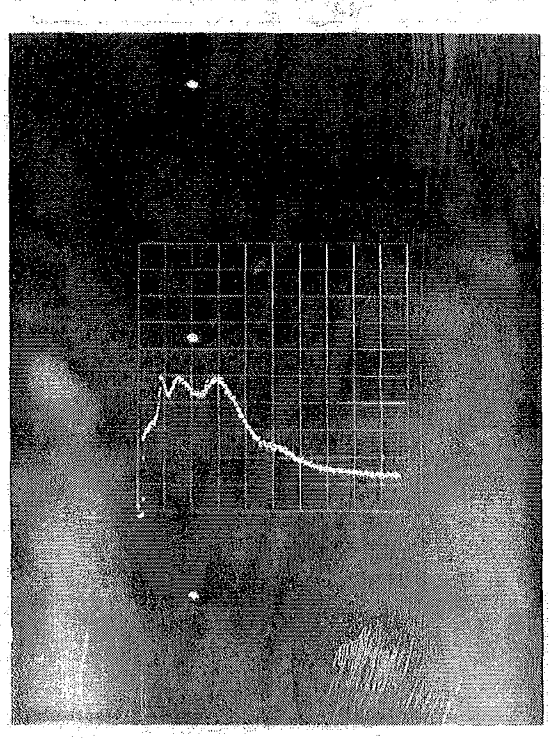



should show energy on this axis for clarity

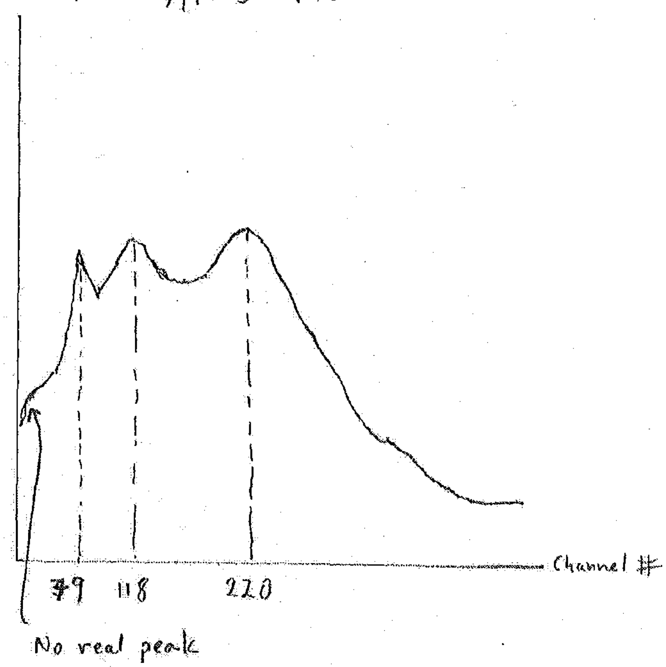


#2
90° Cu scatterer





# 3
90° Al scatterer



# Data Table

| Run | Separation Angle | Duration (hrs.) | Av. Singles rate (per min.) | # of Coincidences | Coincidence rate (per min.) |
|---|---|---|---|---|---|
| 1. | 0° | 69.58 | $1.36 \times 10^4$ | 15,983 | $3.816 \pm .030$ |
| | 90° | 24.00 | $1.39 \times 10^4$ | 22,721 | $7.889 \pm .052$ |
| Background | 0° | 119.25 | | 84 | $0.012 \pm .001$ |
| " | 90° | 72.00 | | 35 | $0.008 \pm .001$ |
| 2. | 180° | 48.03 | $1.38 \times 10^4$ | 13,883 | $4.817 \pm .041$ |
| | 270° | 67.83 | $1.30 \times 10^4$ | 39,992 | $9.826 \pm .049$ |
| | 330° | 25.17 | $1.18 \times 10^4$ | 9,327 | $6.710 \pm .069$ |
| | 300° | 23.85 | $1.30 \times 10^4$ | 14,221 | $9.938 \pm .083$ |
| 3. | 0° | 21.88 | $1.47 \times 10^4$ | 10,413 | $2.475 \pm .024$ |
| | 90° | 23.53 | $1.39 \times 10^4$ | 8,242 | $5.837 \pm .064$ |
| 4. | 0° | 23.58 | $0.99 \times 10^4$ | 3,412 | $2.411 \pm .041$ |
| | 90° | 20.19 | $0.96 \times 10^4$ | 6,763 | $5.582 \pm .068$ |
| Background | 0° | 92.52 | | 2,007 | $0.362 \pm .008$ |
| " | 90° | 24.20 | | 496 | $0.342 \pm .015$ |

These bring up a question of [illegible]

## Coincidence ratio (Background corrected) $\perp / \parallel$

1. $2.072 \pm .030$
2. $2.043 \pm .028$
3. $2.367 \pm .049$
4. $2.557 \pm .075$

[The error bars were determined using $E = \frac{1}{\sqrt{N}}$ where $N$ is the total number of counts.]

Where did you find such long times? Note the drift between runs at the same angle!

The data:

A look at the data table shows several interesting phenomena:

-For runs 1 and 2 the ratio ⊥/∥ is around 2.05, while for run 3 it is 2.37, and for run4 it is 2.56.

- The background rate jumped more than one order of magnitude from run 1 to run 4. *What was your procedure to obtain a background rate?*

- The average singles rate dropped significantly in run 4.

- There is a significant change in coincidencerrates between the various runs.

- The coincidence rate for 300° (60°) is higher than for that at 270° (90°).

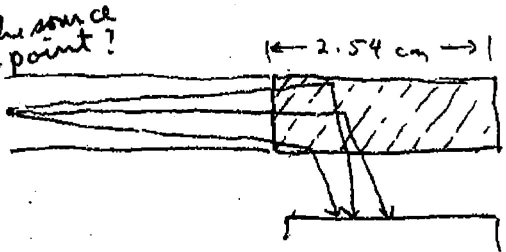
The pertinant conditions under run 1 were: counters set at (1) 3.68 cm and (2) 3.43 cm from the scatterer, wide windows on the SCA's,aa 10 nsec resolving time on the coincidence counter, and the source set at 15.6 on the adjuster (below center). *What is below center? where is the source*

In run 2 we moved counter 1 up to 3.43 cm, equal with counter 2. While not changing the singles rate much, this upped the coincidence rate by about 25%. So evidently the coincidence rate is sharply affected by asymmetries. *(or window drift, etc, etc.)* *is this a verb*

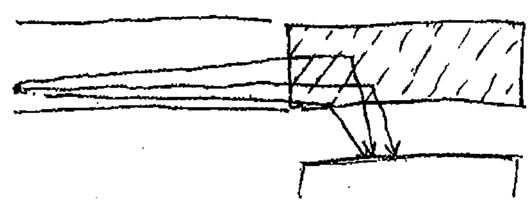
Between runs 2 and 3 we experimented with the source position, finally moving it to 20.3; this was as close to being centered as possible. The result was a decrease in the coincidence rate and an increase in the ratio! To understand this we must look at the situation in more detail. *What is centered?*

*is the source a point?*

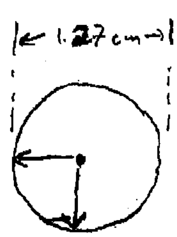
a.



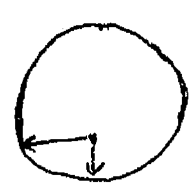

b.



c.




d.



As mentioned before, the mean free path for 511 KeV photons in Cu is 1.35 cm, and after scattering this is even smaller. A look at fig. (a) shows that for a photon coming into the scatterer at $0^o$, the path it must traverse is generally on the order of, or greater than, the mean free path. Any path above center must be even longer, and any path below center must have a corresponding path above center on the opposite side. So the paths around $0^o$ are by far? the most likely to be picked up as a coincidence.

In fig. (b), there is evidently a much smaller angular range for possible coincidences, but the path to be traversed is much shorter in this case. The latter effect will outweigh the former because (from the preceding paragraph) the effective angular range for (a) is probably not much greater than that for (b). So the coincidence rate should be higher for (b).

To get a rough estimate of the difference, assume the photon comes in at $0^o$ in both cases, and that they go the same distance horizontal difference. Then the ratio of case (b) to (a) only depends on the distance traveled after scattering. Assume the vertical path in (a) is .64 cm, and that in (b) is .32 cm. If the mean free path for 255 KeV photons in Cu is 1.07, the ratio is $r=\exp(.32/1.07)= 1.35$. The ratio for the coincidence rates between (a) and (b) is then $r^2=1.82$. From the data table this ratio is, for $\parallel$, 1.95; and for $\perp$, 1.68. Considering the inexactitudes that went into getting the estimate, these values are in pretty good agreement.

Fig.'s (c) and (d) are cross sections of the scatterer, the dot representing the $0^o$ beam line for fig.'s (a) and (b) respectively. Clearly, in fig. (c), $0^o$ and $90^o$ scattering involve traversing the same path length. In fig. (d), however, there is a relatively large difference in the path lengths. Thus we should expect, in this case, that it should be relatively harder to get a coincidence count in the perpendicular orientation, as is in fact the case.

For run 4 we narrowed the window width on the SCA's, as already described, and widened the resolving time on the coincidence counter to 0.5 $\mu$sec.

The widened resolving time resulted in an increased accidental rate; increasing the resolving time from 10 nsec to 0.5 $\mu$sec, a fiftyfold increase, yielded an increased accidental rate of 30 and 43 times the previous values. This makes sense, since the accidental rate should be proportional to the resolving time (as shown earlier), while narrowing the windows should cut into this somewhat. expt'l or theor?

The narrower window width cut down the singles rate from an average of $1.43\times10^4$/min. in run 3 to around $.97\times10^4$/min. in run 4, a decrease of about 32%; however the coincidence rates only went down by 17% and 11% for $0^\circ$ and $90^\circ$ respectively. This indicates that most of our coincidence events took place within the narrow ~23KeV energy range defined by our windows.

The high ratio measured for run 4 is due, in part, to to the fact that the narrow window width effectively decreased the angular spread for acceptable events, thus bringing the ratio up (see the theory section). However, it is still higher than the expected value. A suggestion for further exploration of this would be to do a run with narrow windows at a small resolving time, to separate the two effects. (Time limitations prevented us from trying this.)

The coincidence rate should be a smoothly varying function of the relative orientation of the counters, reaching a maximum at $90^\circ$. Our $330^\circ$ ($30^\circ$) measurement agrees well with this, but the $300^\circ$($60^\circ$) does not. In fact it is higher than the $90^\circ$ measurement, although they are within the error bars of each other. However, not having done the finite geometry problem for $60^\circ$, it is possible that this measurement is really not too far off.

Conclusion:

Due to geometrical considerations, i.e. source and counter placement, runs 3 and 4 must be taken as the most significant tests of the theory. The results of run 3, with $r=2.367 \pm .049$, agrees best with the predicted value of $\sim 2.3$. The result of run 4, $r=2.557 \pm .075$, is, however, slightly ~~significantly~~ higher than the predicted value of $\sim 2.35 \pm .10$. Is $2.56 \pm 0.1$ statistically different than $2.35 \pm 0.1$ ?

It seems to me that the strategy that you adopted of counting for very long times meant that you were unable to get in enough runs to systematically test any of the particular hypotheses that you generated. In the case of the narrow coincidence window, your sensitivity to drift of the delay during a long counting run could easily undercut the apparent statistical accuracy gained. ~~This is~~ With the wide window, accidentals were rather high. Shorter runs (several per lab period, for example) would allow checking the reproducibility of the results for a given setup, and the more systematic variation of things like source height and coincidence window width, as well as angle. WJS.