Groups and Representations II: Problem Set 2

Due Monday, February 19

Problem 1:

Show that a group G (for which the convolution product is defined) is commutative iff the convolution product is commutative.

Problem 2:

For a representation V of $G = SU(2)$, show that the multiplicity of the n'th irreducible representation in V is given by the coefficient of z^{n+1} in

$$(z - z^{-1})\chi_V(z)$$

Problem 3:

Using characters, find the decomposition into irreducibles of the tensor product $V_n \otimes V_m$ of irreducible representations of $SU(2)$.

Problem 4:

Let $G = SU(n)$, and T the subgroup of diagonal elements. Consider the map

$$\pi : G/T \times T \to G$$

given by $\pi(gT, t) = gtg^{-1}$ What is the Weyl group W in this case? Show that this map is a $|W|$-fold covering away from a locus in G of dimension less than $dim\ G$. What is this locus of points in G where π is not a $|W|$-fold covering?

Problem 5:

For $G = SO(2n)$, identify a maximal torus and the positive roots. Give an explicit version of the Weyl integral formula in this case as an integral over this maximal torus.

Problem 6: For $G = SO(3)$, identify a maximal torus T, the space G/T, and the Weyl group W. Give an explicit construction of the irreducible representations of G, compute their characters, and use the Weyl integration formula to show that they are orthonormal.