Quantum Theory, Groups and Representations: Errata

Peter Woit
Department of Mathematics, Columbia University
woit@math.columbia.edu

January 9, 2024

The following are errata for the version of the book published by Springer. These may or may not have yet been fixed in versions of the book available from Springer.

Page viii:
To the list of those helpful with suggestions for improving the text, add Alain Bossavit, Alex Gezerlis, Eduard Sackinger, Lasse Schmieding, John Stroughair and Jieyan Zhu.
Page 8:
The first line of the equation at the bottom of the page would more clearly be written as

$$g_1 \cdot (g_2 \cdot f)(x) = (g_2 \cdot f)(g_1^{-1} \cdot x)$$

Page 11:
Last paragraph. Replace "an action of G of our physical system" by "an action of G on our physical system.
Page 31:
The σ in the last equation should be in bold-face.
Page 64:
In the figure, replace "x" with "X".
Page 85:
Last equation should be

$$\Phi \begin{pmatrix} \alpha & \beta \\ -\overline{\beta} & \overline{\alpha} \end{pmatrix} = \begin{pmatrix} \mathrm{Re}(\alpha^2 - \beta^2) & \mathrm{Im}(\alpha^2 + \beta^2) & -2\,\mathrm{Re}(\alpha\beta) \\ -\mathrm{Im}(\alpha^2 - \beta^2) & \mathrm{Re}(\alpha^2 + \beta^2) & 2\,\mathrm{Im}(\alpha\beta) \\ 2\,\mathrm{Re}(\alpha\overline{\beta}) & 2\,\mathrm{Im}(\alpha\overline{\beta}) & |\alpha|^2 - |\beta|^2 \end{pmatrix}$$

Page 100: The first itemized section should read

- One can check that it satisfies

$$u_+(R\mathbf{x}) = \Omega u_+(\mathbf{x})$$

where $R = \Phi(\Omega)$ is the rotation corresponding to an $SU(2)$ element

$$\Omega = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{-i\phi}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

$u_+(\mathbf{x})$ is determined by setting it to be $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ at the North pole, and defining it at other points $\mathbf{x}$ on the sphere by acting on it by the element Ω which, acting on vectors by conjugation (as usual using the identification of vectors and complex matrices), would take the North pole to $\mathbf{x}$.

The formula for $u_-(\mathbf{x})$ should be

$$u_-(\mathbf{x}) = \frac{1}{\sqrt{2(1+x_3)}} \begin{pmatrix} -(x_1 - ix_2) \\ 1 + x_3 \end{pmatrix} = \begin{pmatrix} -e^{-i\phi}\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}$$

Page 106:
The first equation on the page should be

$$\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \in SU(2) \to \begin{pmatrix} \cos 2\theta & \sin 2\theta & 0 \\ -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \in SO(3)$$

Page 112:
The action on coordinates at the bottom of the page should be by g^{-1} rather than g^T. This affects the formulas on the next page, since it this changes the Lie algebra action from X^T to $-X$, changing the sign for X_1, X_3, but not for X_2. Now it is the z_2^n rather than the z_1^n that are the explicity highest weight vectors.
Page 116:
As in the $SU(2)$ case the action on coordinates should be by g^{-1} rather than g^T. Here this does not affect the results of the calculation, since for elements X of an orthogonal Lie algebra, $X_T = -X$. In particular, in the formula for $\rho'(X)f$, the second e^{tX} should be e^{-tX}.
Page 119:
The first matrix equation on the page should be

$$\begin{pmatrix} \frac{\partial}{\partial r} \\ \frac{\partial}{\partial \theta} \\ \frac{\partial}{\partial \phi} \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ r\cos\theta\cos\phi & r\cos\theta\sin\phi & -r\sin\theta \\ -r\sin\theta\sin\phi & r\sin\theta\cos\phi & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_3} \end{pmatrix}$$

Page 119:
The last equation in the page should be

$$L_3 = i\rho'(l_3) = i\left(x_2 \frac{\partial}{\partial x_1} - x_1 \frac{\partial}{\partial x_2}\right) = -i\frac{\partial}{\partial \phi}$$

Page 151: The integrals in the last displayed equation should be integrals over t, not over ω.

Page 159:
The last equation in the page should be

$$A^t T : f \in \mathcal{S}(\mathbf{R}) \to (A^t T)[f] = T[Af] \in \mathbf{C}$$

Page 168:
The reference to equation 4.7 in the middle of the page should be to equation 4.6.
Page 173:
The equation in the middle of the page should be

$$\psi(q_0, 0) = \frac{1}{\sqrt{2\pi}} e^{ik' q_0}$$

Page 176:
The last equation in the page should be

$$\widehat{U}(\omega, k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left(\frac{1}{\sqrt{2\pi}} e^{-i \frac{1}{2m} k^2 t} \right) e^{i\omega t} dt = \delta(\omega - \frac{1}{2m} k^2)$$

Page 177:
The equation above equation 12.12 should be

$$U_+(t, q_t - q_0) = \lim_{\epsilon \to 0^+} \left(\frac{1}{2\pi} \right)^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{i}{\omega' - \frac{1}{2m} k^2 + i\epsilon} e^{-i\omega' t} e^{ik(q_t - q_0)} d\omega' dk$$

Page 177:
After equation 12.16, should be "$\widehat{D}$ is zero", not "$\widehat{G}$ is zero".
Page 179:
The equation for $\widehat{U}(\omega, k)$ should be

$$\widehat{U}(\omega, k) = \frac{i}{2\pi} \widehat{G}_+(\omega, k)$$

Page 195:
Next to last line, the equation should be $\omega(v, v') = -\omega(v', v)$
Page 221:
Equation 16.14 should be

$$-\mathbf{q} \cdot A\mathbf{p} = -\mathbf{p} \cdot A^T \mathbf{q}$$

Page 247:
Second part of equation 19.1 should be

$$\Gamma'_S(p_2) = -iP_2 = -\frac{\partial}{\partial q_2}$$

Page 288:
Last two equations should be

$$q(0) = c_+ + c_- = 2\,\mathrm{Re}(c+), \quad p(0) = im\omega(c_+ - c_-) = -2m\omega\,\mathrm{Im}(c_+)$$

$$c_+ = \frac{1}{2}q(0) - i\frac{1}{2m\omega}p(0)$$

Page 289:
First two equations should be

$$q(t) = \left(\frac{1}{2}q(0) - i\frac{1}{2m\omega}p(0)\right)e^{i\omega t} + \left(\frac{1}{2}q(0) + i\frac{1}{2m\omega}p(0)\right)e^{-i\omega t}$$

$$p(t) = \left(\frac{im\omega}{2}q(0) + \frac{1}{2}p(0)\right)e^{i\omega t} + \left(\frac{-im\omega}{2}q(0) + \frac{1}{2}p(0)\right)e^{-i\omega t}$$

Page 290:
Last equation in section 22.1 should not have an $\hbar$ in the exponent
Page 303:
The equation in the middle of the page should be

$$\langle\delta_{w_1}|\delta_{w_2}\rangle = e^{\overline{w}_2 w_1}$$

Page 304:
In the equation in the middle of the page a z should be a w.
Page 310:
The equation for the inverse Bargmann transform $\mathcal{B}^{-1}(\phi)$ should have an extra factor of $\frac{1}{\pi}$ on the right-hand side.
Page 311:
The equation at the top of the page should have an extra factor of $\frac{1}{\pi}$ on the right-hand side.
Page 314
Last of three equations at the top of the page should be

$$\{\overline{z}^2, z^2\} = -4iz\overline{z}$$

Page 315
First equation in the text should be

$$\{\overline{z}^2, z^2\} = -4iz\overline{z}$$

Page 324:
The last equation in chapter 24 should be

$$H_r = a_r^\dagger a_r + \frac{1}{2} = (\cosh(2r)a^\dagger a + \frac{1}{2}\sinh(2r)(a^2 + (a^\dagger)^2) + \sinh^2 r + \frac{1}{2}$$

Page 347:
In the last equation of section 26.2, a right parenthesis should be a right bracket (}).

Page 352:
In the equation for $\Gamma'(u^-, 0)$ the notation $\widehat{u}_j$ should be explained as meaning to drop the term.
There is an excess = sign in equation 26.16.
Page 356:
Near middle of page, replace $\mathcal{H}_B \otimes \mathcal{F}_d^+$ by $\mathcal{H}_B \otimes \mathcal{F}_1^+$.
Page 361
The equations for the commutators near the top of the page should be

$$[AB, C] = A[B, C] + [A, C]B$$

and

$$[AB, C] = A[B, C]_+ - [A, C]_+ B$$

Page 414
The equation in the middle of the page should be

$$H|n_B, n_F\rangle = (n_B + n_F)\hbar\omega|n_B, n_F\rangle$$

Page 448
In the first equation of section 36.1.1, $\mathcal{H}_{\text{‘}}$ should be $\mathcal{H}_1$.
Page 454 The third equation from the bottom should read

$$[a(p_j), a(p_k)^\dagger] = \delta_{jk}$$

Page 462
The last equation on the page should be

$$A(p_j, t) = e^{i\frac{p_j^2}{2m}t} A(p_j, 0)$$

Page 463
In section 36.6, the reference to “Three books” should be “Two book”.
Page 517
The second paragraph is wrong. Replace with

This is no longer true for $SL(2, \mathbf{C})$. Conjugation by a fixed matrix will not change the set of eigenvalues of the matrix, and the two eigenvalues are not necessarily complex conjugates (unlike for the case of $SU(2)$). So such a conjugation cannot change all $SL(2, \mathbf{C})$ matrices to their complex conjugates, since in general (complex) conjugation will change the set of eigenvalues.
Page 521
At bottom of the page, the matrices should be

$$-\frac{1}{2}\gamma_1\gamma_2 = -\frac{i}{2}\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \quad -\frac{1}{2}\gamma_1\gamma_3 = \frac{i}{2}\begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}, \quad -\frac{1}{2}\gamma_2\gamma_3 = -\frac{i}{2}\begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix}$$

Page 522

The second set of equations should be

$$\pi'(l_1) = -\frac{1}{2}\gamma_2\gamma_3,\ \pi'(l_2) = \frac{1}{2}\gamma_1\gamma_3,\ \pi'(l_3) = -\frac{1}{2}\gamma_1\gamma_2$$

Page 523
The first matrix on the page should be

$$-i\begin{pmatrix} x_0 + x_3 & x_1 - ix_2 \\ x_1 + ix_2 & x_0 - x_3 \end{pmatrix}$$

Page 534
The last equation in section 42.2 should be

$$W_0 = -\mathbf{p} \cdot \mathbf{J}, \quad \mathbf{W} = -p_0 \mathbf{J} + \mathbf{p} \times \mathbf{K}$$

Page 539
The equations for W_1, W_2, B_1, B_2, W are missing a factor of $|\mathbf{p}|$
Page 572
The commutators for the boost operators should be

$$[-i\widehat{K}_j, -i\widehat{K}_k] = -\epsilon_{jkl}(-i\widehat{L}_l)$$

Page 600:
Instead of

$$\mathcal{H}_1 \otimes \mathbf{C} = \mathcal{M}^+_{j_r} \oplus \mathcal{M}^-_{j_r}$$

read

$$\mathcal{M} \otimes \mathbf{C} = \mathcal{M}^+_{j_r} \oplus \mathcal{M}^-_{j_r}$$

Page 601:
First line, replace $\alpha_\mu(\mathbf{p})$ by $\alpha = (\alpha_0(\mathbf{p}), \boldsymbol{\alpha}(\mathbf{p}))$.
Replace equation 46.22 by

$$\langle \alpha, \alpha' \rangle = \int_{\mathbf{R}^3} (-\overline{\alpha_0(\mathbf{p})}\alpha_0'(\mathbf{p}) + \overline{\boldsymbol{\alpha}(\mathbf{p})} \cdot \boldsymbol{\alpha}'(\mathbf{p}))d^3\mathbf{p}$$

Replace occurrences of $\mathcal{H}_1^{+\prime}$ and $\mathcal{H}_1^{+\prime\prime}$ with $\mathcal{H}_1'$ and $\mathcal{H}_1''$ respectively.
Page 622:
Last paragraph, replace "$\mathbf{A}_R$ and $\mathbf{A}_R$" by "$\mathbf{A}_L$ and $\mathbf{A}_R$".
Page 642:
Near bottom of page, $\mathfrak{su}(\mathfrak{2})$ should be $\mathfrak{su}(2)$.
Page 640:
In problem 3 of section B.3, "the conjugation action of $SL(2, \mathbf{C})$" should be replaced by "the action of $g \in SL(2, \mathbf{C})$ given by left multiplication by g and right multiplication by $g^\dagger$"
Page 643:
In problem 2 of section B.6, in the last line replace k by k_0.
Page 644:
In problem 2, replace "momentum map" by "moment map".
Page 649
In problem 4, on the second line of the page, delete "unitary".