Groups and Representations II: Problem Set 7

Due Monday, March 24

Problem 1: For $G = SO(2n)$, identify a maximal torus and the positive roots.

Give an explicit version of the Weyl integral formula in this case as an integral over this maximal torus.

Problem 2: For $G = SO(3)$, identify a maximal torus T, the space G/T, and

the Weyl group W. Give an explicit construction of the irreducible representations of G, compute their characters, and use the Weyl integration formula to show that they are orthonormal.

Problem 3: For $G = SU(3)$, explicitly define an infinite sequence of irreducible

representations on spaces of homogeneous polynomials analogous to the irreducible representations of $SU(2)$. Use the Borel-Weil theory to describe these geometrically and relate them to their highest weights. Use the Weyl dimension formula to compute the dimensions of these representations.

Problem 4: Show that, for connected, compact G, an irreducible representation

V_λ with highest weight λ satisfies

$$(V_\lambda)^* = V_{w_0 \cdot \lambda}$$

where w_0 is the unique element of the Weyl group that interchanges the positive and negative roots.

Problem 5: Sepanski, Exercise 7.33 and Exercise 7.34, parts (1) and (2).

Problem 6: Show that the real Clifford algebra for $\mathbf{R}^3$ is isomorphic to the

sum of two copies of the quaternion algebra, i.e.

$$C(3) = \mathbf{H} \oplus \mathbf{H}$$