Groups and Representations II: Problem Set 6

Due Monday, February 21

Problem 1: Given two finite groups G_1 and G_2, show that all irreducible representations of $G_1 \times G_2$ are of the form $V_1 \otimes V_2$, where V_1 is an irreducible representation of G_1 and V_2 is an irreducible representation of G_2.

Problem 2: Show that the right action of G on itself induces a representation on $Hom_G(V_i, \mathbf{C}(G))$ isomorphic to V_i^*. Hint: Sepanski pages 62 and 63.

Problem 3: Give a detailed proof of Theorems 7.46 and 7.47 in the Sepanski notes (i.e. that the induced representation is on the sections of an associated bundle, and Frobenius reciprocity)

Problem 4: Given a finite-dimensional representation V, one can form representations S^2V and Λ^2V by taking the symmetric and antisymmetric parts of the tensor product $V \otimes V$. Show that

$$\chi_{S^2V} = \frac{1}{2}(\chi_V(g)^2 + \chi_V(g^2))$$

and

$$\chi_{\Lambda^2V} = \frac{1}{2}(\chi_V(g)^2 - \chi_V(g^2))$$

and thus

$$V \otimes V = S^2V \oplus \Lambda^2V$$

Problem 5: Let $G = SU(n)$, and T the subgroup of diagonal elements. Consider the map

$$\pi : G/T \times T \to G$$

given by $\pi(gT, t) = gtg^{-1}$ What is the Weyl group W in this case? Show that this map is a $|W|$-fold covering away from a locus in G of dimension less than $dim\ G$. What is this locus of points in G where π is not a $|W|$-fold covering?

Problem 6: Find explicitly the maximal torii for the 4 classes of classical groups.