Groups and Representations II: Problem Set 1

Due Tuesday, January 31

Problem 1: Prove that the Schrödinger representation of the Heisenberg group on $L^2(\mathbf{R})$ is irreducible.

Problem 2: Using the definition of the Bargmann-Fock space given in class, show that the operators $a^\dagger$ and a are adjoint operators on this space.

Problem 3: Show that the center $Z(\mathfrak{h})$ of the universal enveloping algebra $U(\mathfrak{h})$ is isomorphic to the algebra of polynomials in the generator Z (the one that commutes with X, Y).

Problem 4: Show that the real polarizations of $V = \mathbf{R}^{2n}$ with a symplectic form S are parametrized by $U(n)/O(n)$.

Problem 5: Show that the positive complex polarizations of $V = \mathbf{R}^{2n}$ with a symplectic form S are parametrized by $Sp(2n, \mathbf{R})/U(n)$.