

LIE GROUPS AND REPRESENTATIONS, SPRING 2016
Problem Set 2

Due Monday, February 8

Problem 1: Let \mathfrak{h} be a Cartan subalgebra of a complex simple Lie algebra \mathfrak{g} , and $\mathfrak{n}_+, \mathfrak{n}_-$ subalgebras corresponding to the span of the root spaces of the positive and negative roots respectively (for some choice of this decomposition).

- Show that \mathfrak{n}_+ and \mathfrak{n}_- are nilpotent Lie algebras.
- Show that the Borel subalgebra

$$\mathfrak{b} = \mathfrak{h} \oplus \mathfrak{n}_+$$

is a solvable Lie algebra.

Problem 2: Find a choice of Cartan subalgebra, of simple roots, and of positive roots for the Lie algebra $\mathfrak{sl}(n, \mathbf{C})$. Find the root spaces for the positive roots. Compute the Cartan matrix.

Problem 3: Find a choice of Cartan subalgebra, of simple roots, and of positive roots for the Lie algebra $\mathfrak{so}(2n, \mathbf{C})$. Find the root spaces for the positive roots. Compute the Cartan matrix.

Problem 4: Find an explicit isomorphism of the Lie algebras $\mathfrak{sl}(4, \mathbf{C})$ and $\mathfrak{so}(6, \mathbf{C})$.