# Lie Groups and Representations, Spring 2016
## Final Problem Set

**Due Wednesday, May 4**

**Problem 1:** Prove the classification theorem for complex Clifford algebras, i.e.

$$\mathrm{Cliff}(2k, \mathbf{C}) = M(2^k, \mathbf{C}), \mathrm{Cliff}(2k+1, \mathbf{C}) = M(2^k, \mathbf{C}) \oplus M(2^k, \mathbf{C})$$

for $k = 0, 1, 2, \ldots$.
Show that the even part of the Clifford algebra $\mathrm{Cliff}(n, \mathbf{C})$ is isomorphic to the Clifford algebra $\mathrm{Cliff}(n-1, \mathbf{C})$.

**Problem 2:** Given a construction of the half-spin representations in even dimensions, construct the spin representation in odd dimensions. Find its highest weight, and show that its character is that given by the Weyl character formula.

**Problem 3:** Show that elements of $G = SL(2, \mathbf{R})$ can be uniquely parametrized by the Iwasawa decomposition $G = KAN$ given in class.

**Problem 4:** Use the result of Problem 3 to give coordinates on the group $G$, and to find formulas for the left and right action by basis elements of $\mathfrak{sl}(2, \mathbf{R})$ as vector fields (first-order differentiation operators). Use these to find a formula for the Casimir operator as a second-order differential operator.