Lie Groups and Representations, Spring 2012: Syllabus

- Lie Groups and Lie Algebras: review of fall semester and survey of classification results
- Generalities about representations
  - Complete reducibility, Lie algebra cohomology
  - Induced representations, Frobenius reciprocity
  - Verma modules
  - Peter-Weyl theorem
- Finite-dimensional representations and highest-weight theory
  - Finite dimensional representations of $\mathfrak{sl}(2, \mathbf{C})$
  - Highest-weight theorem via Verma modules
  - Examples, $\mathfrak{sl}(3, \mathbf{C})$ and $SU(3)$
  - Borel-Weil theorem
  - Weyl character formula, dimension and multiplicity formulae
  - Conjugacy classes in semi-simple Lie algebras
  - Harish-Chandra homomorphism, the infinitesimal character
  - $\mathfrak{n}$-cohomology
  - Borel-Weil-Bott theorem
- The Spinor Representation
  - Spin(2n) as a double cover of SO(2n)
  - The Clifford Algebra, Canonical Anticommutation Relations
  - The Dirac operator and Dirac cohomology
- Hamiltonian mechanics, symplectic geometry, geometric quantization and the orbit method
- The Metaplectic Representation
  - The Heisenberg algebra and group, Canonical Commutation Relations
  - Stone-von Neumann Theorem.
  - The Metaplectic double cover of Sp(2n) and the Metaplectic Representation
- Representations of non-compact semi-simple groups: generalities
- Representations of $SL(2, \mathbf{R})$