Groups and Representations, Spring 2012
Problem Set 3
Due Monday, February 27

Problem 1: Consider the representation V of $\mathfrak{sl}(n, \mathbf{C})$ on symmetric powers $S^k(\mathbf{C}^n)$.
a) Identify the weights of this representation and the corresponding weight spaces.
b) Show that this representation contains a unique highest weight. What is it explicitly?
c) Show that this representation is the space of holomorphic sections of a line bundle (which one?).
d) What is the infinitesimal character of this representation?

Problem 2: For the case of $G = SU(3)$, identify explicitly the set of integral weights λ such that

$$H^{0,i}(G/T, L_\lambda) = 0$$

for all i.
Consider the highest weight of the adjoint representation of $SU(3)$. Letting the Weyl group act on this gives a of six different weights λ_j. Compute the cohomology

$$H^{0,i}(G/T, L_{\lambda_j})$$

for all choices of i, j (i.e. what is its dimension?, what is it as an $SU(3)$ representation?)

Problem 3: For a good detailed exposition of Lie algebra cohomology and Kostant's theorem, try reading through
`http://www.math.rutgers.edu/~ goodman/pub/symmetry/appe.pdf`
Work out solutions to the following exercises found there:
E 1.6 Problems 6 and 8
E 2.7 Problem 2