

# Magnetized quark and strange quark matter in the spherical symmetric space–time admitting conformal motion

C. Aktas · İ. Yilmaz

Received: 10 January 2007 / Accepted: 6 March 2007 / Published online: 11 April 2007  
© Springer Science+Business Media, LLC 2007

**Abstract** In this paper, we have examined magnetized quark and strange quark matter in the spherical symmetric space–time admitting one-parameter group of conformal motions. For this purpose, we have solved Einstein’s field equations for spherical symmetric space–time via conformal motions. Also, we have discussed the features of the obtained solutions.

**Keywords** Magnetized quark matter and magnetized strange quark matter · Conformal motion

## 1 Introduction

The idea of primordial magnetism is attractive because it can potentially explain all the large-scale fields seen in the universe today, especially those found in high-red-shift protogalaxies. Any cosmological magnetic field, however, must be consistent with a number of astrophysical constraints. These come from primordial nucleosynthesis [1, 2], and the high isotropy of the Cosmic Microwave Background (CMB) radiation [3–21]. The latter constrains the current strength of a large-scale homogeneous  $B$ -field below  $\sim 10^{-9} G$  [8], although tangled random fields on small scales could reach up to  $\sim 10^{-6} G$ .

---

C. Aktas  
Department of Mathematics, Arts and Sciences Faculty, Çanakkale Onsekiz Mart University,  
Terzioğlu Campus, 17100 Çanakkale, Turkey  
e-mail: canaktas@comu.edu.tr

İ. Yilmaz (✉)  
Department of Physics, Arts and Sciences Faculty, Çanakkale Onsekiz Mart University,  
Terzioğlu Campus, 17100 Çanakkale, Turkey  
e-mail: iyilmaz@comu.edu.tr

Magnetic fields are a widespread and significant component of the Universe [22–27]. The Milky Way and many other spiral galaxies possess magnetic fields with strengths of order a few micro Gauss, that are coherent over the plane of the galactic disc. Magnetic fields are also common property of galaxy clusters, extending well beyond their core regions. The strengths of the ordered magnetic fields in the intracluster medium exceed those typically associated with the interstellar medium of the Milky Way, suggesting that galaxy formation, and even cluster dynamics, could be influenced by magnetic forces. Furthermore, reports of Faraday rotation in high-redshift, Lyman- $\alpha$  absorption systems suggest that dynamically significant magnetic fields could be present in protogalactic condensations.

Despite their widespread presence, however, the precise origin of any large-scale cosmological magnetic field is still a mystery, and its likelihood or necessity remains a subject of debate [28–32].

The relation between strong magnetic fields and dense matter is a subject that have attracted so much attention recently, especially after the observations of peculiar X-ray emission from anomalous X-pulsars (AXPs) and low energy  $\gamma$ -ray radiation from soft gamma-ray repeaters (SGRs). The central engine of these radiations is believed to be a neutron star or ‘quark star’ endowed with a magnetic field larger than  $10^{13.5}$  G. The X-ray binaries dubbed as Galactic Black Hole Candidates have also been recently suggested as possessing a strange star as its primary [33–35].

Recently, quark-gluon plasma is created as a perfect liquid in the Brookhaven National Laboratory [36–38]. So, we shall consider quark-gluon plasma in the form of perfect fluid.

The possibility of the existence of quark matter dates back to early seventies. Itoh [39], Bodmer [40] and Witten [41] proposed two ways of formation of quark matter: the quark-hadron phase transition in the early universe and conversion of neutron stars into strange ones at ultrahigh densities.

In the theories of strong interaction, quark bag models suppose that breaking of physical vacuum takes place inside hadrons. As a result, vacuum energy densities inside and outside a hadron become essentially different and the vacuum pressure on the bag wall equilibrates the pressure of quarks, thus stabilizing the system. If the hypothesis of the quark matter is true, then some neutron stars could actually be strange stars, built entirely of strange matter [42,43]. For a review of strange star properties, see [44].

Typically, quark matter is modeled with an equation of state (EOS) based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In the framework of this model the quark matter is composed of massless u, d quarks, massive s quarks and electrons. In the simplified version of the bag model, assuming quarks are massless and non-interacting, we then have quark pressure  $p_q = \rho_q/3$  ( $\rho_q$  is the quark energy density); the total energy density is

$$\rho = \rho_q + B \quad (1)$$

but total pressure is

$$p = p_q - B. \quad (2)$$

One therefore get equation of state for strange quark matter [45,46]

$$p = \frac{1}{3}(\rho - 4B), \quad (3)$$

where  $B$  is the difference between the energy density of the perturbative and non-perturbative QCD vacuum (the bag constant). Equation (3) is essentially the equation of state of a gas of massless particles with corrections due to the QCD trace anomaly and perturbative interactions. These corrections are always negative, reducing the energy density at given temperature by about a factor of two [47]. For quark stars obeying the bag model equation of state (3) the Chandrasekhar limit has been evaluated, from simple energy balance relations, in [48]. In addition to fundamental constants, the maximum mass also depends on the bag constant.

Recently, Dey et al. [49] have obtained new sets of EOSs for strange matter based on a model of interquark potential which has the following features: (a) asymptotic freedom, (b) confinement at zero baryon density and deconfinement at high baryon density, (c) chiral symmetry restoration and (d) gives stable uncharged  $\beta$ -stable strange matter. These EOSs have later been approximated to a linear form by [50] which has the from,

$$p = \epsilon(\rho - \rho_0), \quad (4)$$

where  $\rho_0$  denotes the energy density at zero pressure and  $\epsilon$  is a constant.

Also, recent investigations show that a number of pulsars like Her X-1 [51], 4U 1820-30 [49,52], SAX J 1808.4-3658 [53], 4U 1728-34 [54], PSR 0943+10 [55,56] and RX J185635-3754 [53,57], earlier thought to be neutron stars, are actually good strange star candidates.

On the other hand, it is well-known that strong magnetic fields, as large as  $B \sim 10^{12} - 10^{14}$  G, exist in the surface of neutron stars [58–63], while in magnetars they are in the range  $B \sim 10^{14} - 10^{15}$  G, and perhaps as high as  $10^{16}$  G [64]. It is presumed from the virial theorem [65] that the interior field in neutron stars could be as high as  $10^{18} - 10^{19}$  G. If quark stars are self-bound rather than gravitational-bound objects, the previous upper limit that has been obtained by comparing the magnetic and gravitational energies could go even higher.

Because of the anti-screening, this condensate does not give a mass to the  $\tilde{Q}$  photon, but instead amplifies the applied rotated magnetic field. This means that in the CS a sort of anti-Meissner effect takes place. This effect can be also of interest for astrophysics since once the core of a compact star becomes color superconducting, its internal magnetic field can be boosted to values higher than those found in neutron stars with cores of nuclear matter. This effect could open a new window to differentiate a neutron star made up entirely of nuclear matter from one with a quark matter core.

Thus, investigating the effect of strong magnetic fields in color superconductivity is of interest for the study of compact stars in astrophysics.

Also the experimental measurement and theoretical investigation of electromagnetic observables in heavy-ion collision constitutes one of the most promising and exciting fields in high energy physics. This promising and exciting observables indicate nothing but the emission of high energy photons and leptons [66–68] and these particles have a large mean free path due to the small cross section for electromagnetic

interaction in the plasma. These particles carry the whole informations about the existence of the plasma and it is moreover true that over a large range of expected plasma temperature, its radiation can be observed throughout its evolution. So they are considered to be the good signature for the formation of QGP.

In this study we will examine magnetized quark matter in the spherical symmetric space–time by using conformal collineation.

General Relativity provides a rich arena to use symmetries in order to understand the natural relation between geometry and matter furnished by the Einstein equations. Symmetries of geometrical/physical relevant quantities of this theory are known as collineations. The most useful collineations is conformal killing vectors.

Conformal collineation is defined by,

$$\mathcal{L}_\xi g_{ik} = 2\psi g_{ik}, \quad \psi = \psi(x^i) \quad (5)$$

where  $\mathcal{L}_\xi$  signifies the Lie derivative along  $\xi^i$  and  $\psi(x^i)$  is the conformal factor. In particular,  $\xi$  is a special conformal killing vector (SCKV) if  $\psi_{;ik} = 0$  and  $\psi_{;i} \neq 0$ . Other subcases are homothetic vector (HV) if  $\psi_{;i} = 0$  and  $\psi \neq 0$ , and Killing vector (KV) if  $\psi = 0$ . Here  $(;)$  and  $(,)$  denote the covariant and ordinary derivatives, respectively.

Conformal killing Vectors provide a deeper insight into the space–time geometry and facilitate generation of exact solutions to the field equations.

The study of conformal motions in space–time is physically very important. Because, it can lead to the discovery of conservation laws and devise space–time classification schemes. For example, collineations can be considered as non-Noetherian symmetries and can be associated with constants of motion and, up to the level of conformal killing vectors, they can be used to simplify the metric [69]. Affine Collineations are related to conserved quantities [70] (a result used to integrate the geodesics in FRW space–times), Ricci Collineations are related to the conservation of particle number in FRW space–times [71] and the existence of curvature collineations imply conservation laws for null electro-magnetic fields [72].

Also, the existence of a conformal killing vector is closely related to “global” equilibrium properties of perfect fluids. For instance, the conformal symmetry singles out a perfect fluid with the equation of state for radiation.

In other words, for massless particles the condition for global equilibrium requires the quantity  $u^i/T$ , where  $u^i$  is the fluid four-velocity and  $T$  is the fluid temperature, to be a conformal killing vector (CKV). Only the conformal symmetry of an “optical” metric, in which an effective refraction index of the cosmic substratum characterizes specific internal interactions that macroscopically correspond to a negative pressure contribution, may be compatible with the production of entropy [73–75].

Further more, we are able to reduce partial differential equations to ordinary differential equations by using conformal motions.

So, in this paper it is imposed the condition that the space time manifold admits a conformal motion. Because the use of conformal motions, instead of homothetic motions ( $\phi = \text{constant}$ ), allow us to find static and spherically symmetric distributions of matter which may be fitted to the exterior Schwarzschild metric [76]. Also, we have chosen to study spherically symmetric space–times since, other than FRW

space–times, the majority of the remaining conformal motion space–times known to the authors are spherically symmetric.

Conformal collineations have been studied at length by various authors. Herrera et al. [77] have studied conformal collineations, with particular reference to perfect fluids and anisotropic fluids; Duggal and Sharma [78] extend this work to the more general case of a special Affine Collineation  $\xi^i$ ; Coley and Tupper [79] have discussed space–times admitting special conformal killing vector and symmetry inheritance; Mason and Maartens [80] have considered kinematics and dynamics of conformal collineations; Maartens et al. [81] have studied the conformal collineations in anisotropic fluids, in which they are particularly concerned with special conformal killing vectors. Coley and Tupper [82] and Maartens [83] have studied conformal motion in the spherically symmetric space–times. Carot et al. [84] have discussed space–times with conformal motions. Recently, Yilmaz [85, 86] have studied quark matter attached to the string cloud and domain wall. Also Yavuz et al. [87] has studied Strange quark matter attached to the string cloud in the spherical symmetric space–time admitting conformal motion. Mak and Harko [88] have studied charged strange quark matter in the spherical symmetric space–time admitting conformal motion.

The paper is outlined as follows. In Sect. 2, Einstein field equations and their solutions are obtained for magnetized quark matter in the Spherically symmetric space–times by using conformal motions. In the Sect. 3, solutions of the Einstein field equations are obtained for magnetized strange quark matter. In Sect. 4, concluding remarks are given.

## 2 Magnetized quark matter solutions admitting conformal motions

We can examine magnetized quark and strange quark matter due to two reasons.

One is that our present knowledge of QCD at high density indicates that quark matter might be in a color super conducting phase. So the merit of the infinite or super conductivity assumption is that, based on Ohm's law, it can accommodate a zero electric field with non-vanishing spatial currents (i.e.  $h_j^k J_k \neq 0$ ). This means that electric field vanishes as long as the conductivity of the medium is infinite. Thus, at the limit of ideal magnetohydrodynamics, the electric field vanishes in the frame of fluid. On the other hand, zero electric conductivity implies that the spatial currents vanish, even when the electric field is non-zero.

Second is that quark–gluon plasma should satisfy charge neutrality. It means that quark–gluon plasma is electrically neutral. So, quark–gluon plasma should be magnetized due to above two reasons.

The fundamental variables in ideal MHD are the rest-mass density  $\rho$ , pressure  $p$ , four-velocity  $u^i$ , and magnetic field  $h^i$  measured by an observer comoving with the fluid. The ideal MHD condition is written as  $u_i F^{ik} = 0$ , where  $F^{ik}$  is the electromagnetic tensor. The tensor  $F^{ik}$  and its dual in the ideal MHD approximation are given by

$$F^{ik} = \epsilon^{iklm} u_l h_m, \quad (6)$$

$$F_{ik}^* = \frac{1}{2} \epsilon_{iklm} F^{lm} = h_i u_k - h_k u_i, \quad (7)$$

where  $\epsilon_{iklm}$  is the Levi-Civita tensor. The magnetic field measured by a normal observer  $n^i$  is given by

$$H^i \equiv n_k F^{*kj} = \alpha(u^i h^j - h^i u^j), \tag{8}$$

From the definition of  $H^i$  and the antisymmetry of  $F^{ik}$  we have  $n_i H^i = 0 = H^i$ . The relations between  $h^i$  and  $H^i$  are

$$h^i = \frac{H^j u_j}{\alpha}, \tag{9}$$

$$h^i = \frac{1}{\alpha u^i} \left( H^i + H^j u_j u^i \right). \tag{10}$$

Using these variables, the energy-momentum tensor [89] is written as

$$\begin{aligned} T_{ik}^{EM} &= F_{il} F_k^l - \frac{1}{4} g_{ik} F_{ab} F^{ab} \\ &= \left( \frac{1}{2} g_{ik} + u_i u_k \right) h^2 - h_i h_k. \end{aligned} \tag{11}$$

where  $h^2 = h^i h_i$ .

Also, due to behavior of perfect fluid of quark-matter [36–38], the form of the energy momentum tensor of quark matter is taken as

$$T_{ik}^{Quark} = (\rho + p)u_i u_k + p g_{ik} \tag{12}$$

where  $\rho = \rho_q + B$  and  $p = p_q - B$ . Also  $p$  and  $\rho$  are related by equations of state, i.e. Eqs. (3) and (4).

So, for our purposes the total energy-momentum tensor has the form  $T_{ik} = T_{ik}^{Quark} + T_{ik}^{EM}$ , where  $T_{ik}^{Quark}$  is given by Eq. (12) and  $T_{ik}^{EM}$  is given by Eq. (11)

Hence, the total stress-energy tensor for magnetized quark matter becomes

$$T_{ik} = (\rho + p + h^2)u_i u_k + \left( p + \frac{h^2}{2} \right) g_{ik} - h_i h_k. \tag{13}$$

where  $p_{mag} = h^2/2$  is the magnetic pressure and also  $u^i$  is the four velocity  $u^i = \delta_4^i e^{-\nu/2}$ . Also, we can choose magnetic flux in the direction of radial due to  $h_i u^i = 0$ .

Also, Einstein equations can be expressed as

$$R_{ik} - \frac{1}{2} R g_{ik} = 8\pi T_{ik}, \tag{14}$$

Let us consider a static distribution of matter represented by spherical symmetric matter which may be magnetized quark and strange quark matter. We have chosen spherically symmetric metric due to two reasons. One is that geometry of the stars is generally assumed spherically symmetric in the ideal cases. Second is that FRW

universe which describes our universe is also spherically symmetric. Also, we have chosen to study spherically symmetric space–times since, other than FRW space–times, the majority of the remaining conformal motion space–times known to the authors are spherically symmetric. Furthermore, because the use of conformal motions, instead of homothetic motions, allow us to find static and spherically symmetric distributions of matter.

In Schwarzschild coordinates the line element takes the following form:

$$ds^2 = -e^{v(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 d\Omega^2 \tag{15}$$

with

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2, \quad x^{1,2,3,4} \equiv r, \theta, \phi, t$$

Here we shall use geometrized units so that  $8\pi G = c = 1$  and we shall assume the direction of magnetic field in the radial direction. Using the line element Eq. (15) from Eqs. (13) and (14) we get

$$\rho + \frac{h^2}{2} = -e^{-\lambda} \left( \frac{1}{r^2} - \frac{\lambda'}{r} \right) + \frac{1}{r^2} \tag{16}$$

$$p + \frac{h^2}{2} = -e^{-\lambda} \left( \frac{1}{r^2} + \frac{v'}{r} \right) + \frac{1}{r^2} \tag{17}$$

$$p + \frac{h^2}{2} = \frac{e^{-\lambda}}{2} \left( v'' + \frac{v'^2}{2} + \frac{v' - \lambda'}{r} - \frac{v'\lambda'}{2} \right) \tag{18}$$

where primes denote differentiation with respect to  $r$ .

Now we shall assume that space–time admits a one-parameter group of conformal motions Eq. (5), i.e.

$$\mathcal{L}_\xi g_{ik} = \xi_{i;k} + \xi_{k;i} = \psi g_{ik}, \tag{19}$$

where  $\psi$  is an arbitrary function of  $r$ . From Eqs. (15) and (19) and by virtue of spherical symmetry, we get the following expressions

$$\xi^1 v' = \psi, \tag{20}$$

$$\xi^4 = C_1 = \text{const}, \tag{21}$$

$$\xi^1 = \psi r/2, \tag{22}$$

$$\lambda' \xi^1 + 2\xi^1_{,1} = \psi, \tag{23}$$

where a comma denotes partial derivatives. From Eqs. (20)–(23), we get

$$e^v = C_2^2 r^2, \quad (24)$$

$$e^\lambda = \left( \frac{C_3}{\psi} \right)^2, \quad (25)$$

$$\xi^i = C_1 \delta_4^i + (\psi r/2) \delta_1^i, \quad (26)$$

where  $C_2$  and  $C_3$  are constants of integration [77]. **Expression Eqs. (24)–(26) contain all the implications derived from the existence of the conformal collineation.**

Now substituting Eqs. (24) and (25) into Eqs. (16)–(18), we have

$$\rho + \frac{h^2}{2} = \left( \frac{1}{r^2} \right) \left( 1 - \frac{\psi^2}{C_3^2} \right) - \frac{2\psi\psi'}{C_3^2 r} \quad (27)$$

$$-p + \frac{h^2}{2} = \left( \frac{1}{r^2} \right) \left( 1 - \frac{3\psi^2}{C_3^2} \right) \quad (28)$$

$$p + \frac{h^2}{2} = \frac{\psi^2}{C_3^2 r^2} + \frac{2\psi\psi'}{C_3^2 r} \quad (29)$$

from Eqs. (27–29) we get

$$\rho = \frac{1}{2r^2} - \frac{3\psi\psi'}{C_3^2 r} \quad (30)$$

$$p = \frac{1}{r^2} \left( \frac{2\psi^2}{C_3^2} - \frac{1}{2} \right) + \frac{\psi\psi'}{C_3^2 r} \quad (31)$$

$$h^2 = \frac{2}{r^2} \left( \frac{1}{2} - \frac{\psi^2}{C_3^2} \right) + \frac{2\psi\psi'}{C_3^2 r} \quad (32)$$

Using Eqs. (24) and (25), space–time geometry of magnetized quark matter (i.e. line element given by Eq. (15)) becomes

$$ds^2 = C_2^2 r^2 dt^2 - \frac{C_3^2}{\psi^2} dr^2 - r^2 d\Omega^2 \quad (33)$$

If the function  $\psi(r)$  is specified a priori, the problem will be fully determined. According to Eqs. (30)–(32), different solutions can be obtained by specifying the choice of  $\psi(r)$ . In what follows, instead of arbitrarily assuming function  $\psi(r)$ , we shall find some solutions by using Eqs. (30)–(32) from physical considerations depending on dynamical quantities ( $h^2 = 0$ ,  $p = 0$ ).



**Case(i)** If  $\psi = \pm \frac{1}{2}(2C_3^2 + 4r^2C_4)^{1/2}$  then from Eqs. (30)–(32) we get

$$\rho = \frac{1}{2r^2} - \frac{3C_4}{C_3^2} \tag{34}$$

$$p = \frac{1}{r^2} + \frac{3C_4}{C_3^2} \tag{35}$$

$$h^2 = 0 \tag{36}$$

In this case, the line element is as follows

$$ds^2 = C_2^2 r^2 dt^2 - \frac{2C_3^2}{C_3^2 + 2r^2 + C_4} dr^2 - r^2 d\Omega^2 \tag{37}$$

where  $C_4$  integration constant.

**Case(ii)** If  $\psi^2 = \frac{C_3^2}{4} + \frac{C_5}{r^4}$ , then Eqs. (30)–(32) give

$$\rho = \frac{1}{2r^2} + \frac{6C_5}{C_3^2 r^6} \tag{38}$$

$$p = 0 \tag{39}$$

$$h^2 = \frac{1}{2r^2} - \frac{6C_5}{C_3^2 r^6} \tag{40}$$

and the line element is given by

$$ds^2 = C_2^2 r^2 dt^2 - \frac{4C_3^2 r^4}{C_3^2 r^4 + 4C_5} dr^2 - r^2 d\Omega^2 \tag{41}$$

where  $C_5$  is integration constant.

### 3 Magnetized strange quark matter solutions admitting conformal motions

Since strange quark matter, i.e. strange quark stars, are governed by equation of state (Eqs. 3 and 4), we impose these equations for the strange quark matter. Using equations of state (Eqs. 3 and 4) for strange quark matter, we can specify the function  $\psi(r)$  a priori. So, the problem will be fully determined for strange quark matter.

**Case (A)** Strange quark matter in the BAG model:

From Eqs. (3), (30) and (31) we get

$$\psi(r) = \pm \frac{\sqrt{-3C_3^2 Br^4 + 3C_3^2 r^2 + 9C_6}}{3r} \tag{42}$$

Substituting Eq. (42) into Eqs. (30), (31) and (32) we have

$$\rho = \frac{1}{2r^2} + \frac{3C_6}{C_3^2 r^4} + B \quad (43)$$

or

$$\rho_q = \frac{1}{2r^2} + \frac{3C_6}{C_3^2 r^4} \quad (44)$$

$$p = \frac{1}{6r^2} + \frac{C_6}{C_3^2 r^4} - B \quad (45)$$

or

$$p_q = \frac{1}{6r^2} + \frac{C_6}{C_3^2 r^4} \quad (46)$$

and also

$$h^2 = \frac{1}{3r^2} - \frac{4C_6}{C_3^2 r^4} \quad (47)$$

In this case space–time geometry of the strange quark matter or star is given by

$$ds^2 = C_2^2 r^2 dt^2 - \frac{3C_3^2 r^2}{C_3^2 r^2 - C_3^2 B r^4 + 3C_6} dr^2 - r^2 d\Omega^2 \quad (48)$$

**Case (B)** Strange quark matter in the case of linear equation of state:

From Eqs. (4), (30) and (31) we have

$$\psi(r) = \pm \frac{\sqrt{3} \sqrt{3(\epsilon + 1)^2 C_3^2 + 12(\epsilon + 1) C_7 r^{-4/(3\epsilon+1)} - 4\epsilon C_3^2 \rho_0 r^2}}{6\sqrt{\epsilon + 1}} \quad (49)$$

Using Eq. (49) into Eqs. (30)–(32) we get

$$\rho = \frac{(3\epsilon + 1)(\epsilon + 1)C_3^2 + 2(3\epsilon + 1)\rho_0 C_3^2 r^2 + 12(\epsilon + 1)C_7 r^{-4/(3\epsilon+1)}}{(3\epsilon + 1)(\epsilon + 1)C_3^2 r^2} \quad (50)$$

$$p = -\epsilon \frac{-(3\epsilon + 1)(\epsilon + 1)C_3^2 + 2(3\epsilon + 1)\rho_0 C_3^2 r^2 - 12(\epsilon + 1)C_7 r^{-4/(3\epsilon+1)}}{(3\epsilon + 1)(\epsilon + 1)C_3^2 r^2} \quad (51)$$

and

$$h^2 = \frac{(3\epsilon + 1)(1 - \epsilon)C_3^2 - 12(\epsilon + 1)C_7 r^{-4/(3\epsilon+1)}}{(3\epsilon + 1)(\epsilon + 1)C_3^2 r^2} \quad (52)$$

In this case space–time geometry of the strange quark matter or star is as follows

$$ds^2 = C_2^2 r^2 dt^2 - \frac{12(\epsilon + 1)C_3^2}{3(\epsilon + 1)^2 C_3^2 + 12(\epsilon + 1)C_7 r^{-4/(3\epsilon+1)} - 4C_3^2 \rho_0 \epsilon r^2} dr^2 - r^2 d\Omega^2 \tag{53}$$

### 4 Concluding remarks

To understand how a magnetic field can affect the color superconducting pairing it is important to recall that in spin-zero color superconductivity, although the color condensate has non-zero electric charge, there is a linear combination of the photon and a gluon that remains massless.

In this paper, we have studied magnetized quark and strange quark matter in the spherical symmetric space–time admitting one-parameter group of conformal motions. We have obtained the following properties.

$e^\mu$  and  $e^\lambda$  are positive, continuous and non-singular for  $r < r_0$ . In the case (i) of magnetized quark matter, we get only quark matter solution without magnetic field.

In the case (A) of strange quark matter, at the surface of star, as  $p \rightarrow 0$ , we have  $\rho \rightarrow 4B$  (see Eqs. 43 and 45). If we consider that the typical values of the bag constant is of order  $B = 57 \text{ MeV/fm}^3 \approx 10^{14} \text{ g/cm}^3$ , we get the energy density of strange quark stars as  $\rho \approx 4 \times 10^{14} \text{ g/cm}^3$  which coincides with the result of Witten [41].

Also, we get that inside fluid sphere the velocity of sound  $0 \leq v_s \left( = \sqrt{\frac{dp}{d\rho}} \right) \leq 1$  (see Eqs. 54 and 55).

In this case, the gradients  $\frac{d\rho}{dr}$  and  $\frac{dp}{dr}$  should be negative. So, we get the following quantities;

$$\frac{d\rho}{dr} = -\frac{1}{r^3} - \frac{12C_6}{C_3^2 r^5} \tag{54}$$

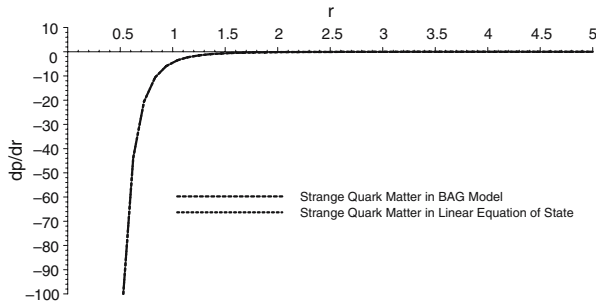
and

$$\frac{dp}{dr} = -\frac{1}{3r^3} - \frac{4C_6}{C_3^2 r^5} \tag{55}$$

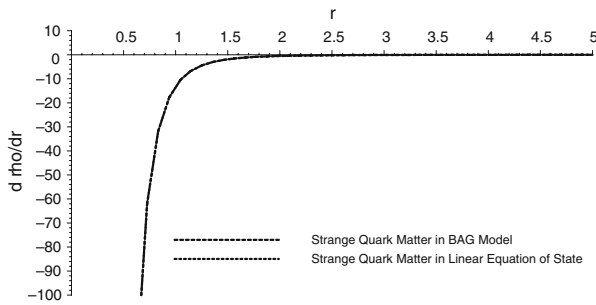
If we set  $C_6 = 0$  and take electric field instead of magnetic field in the case of magnetized strange quark matter in the BAG model, the equations in the our solutions reduce to those of Mak and Harko [88].

In the case (B) of strange quark star, one may obtains the values  $\epsilon = 0.463$ ,  $\rho_0 = 1.15 \times 10^{15} \text{ g/cm}^3$  and  $\epsilon = 0.455$ ,  $\rho_0 = 1.33 \times 10^{15} \text{ g/cm}^3$ , according to B values, respectively. Also, the standard bag model corresponds to  $\epsilon = 0.333$  and  $\rho_0 = 4 \times 10^{14} \text{ g/cm}^3$  (see Figs. 1, 2).

In this case, we get the same geometry for the strange quark matter in the BAG Model and in the case of linear equation of state.



**Fig. 1** The variation of  $dp/dr$  of strange quark matter in the BAG and in the linear model



**Fig. 2** The variation of  $d\rho/dr$  of strange quark matter in the BAG and in the linear model

Also, in this case, because of the gradients  $\frac{d\rho}{dr}$  and  $\frac{dp}{dr}$  should be negative, we get the following quantities;

$$\frac{d\rho}{dr} = -\frac{(1 + 3\epsilon)^2 C_3^2 + 36(\epsilon + 1)C_7 r^{-4/(3\epsilon+1)}}{(1 + 3\epsilon)^2 C_3^2 r^3} \tag{56}$$

and

$$\frac{dp}{dr} = -\epsilon \frac{(1 + 3\epsilon)^2 C_3^2 + 36(\epsilon + 1)C_7 r^{-4/(3\epsilon+1)}}{(1 + 3\epsilon)^2 C_3^2 r^3} \tag{57}$$

These models can be used for non-rotating magnetized strange quark stars. Also, the existence of magnetic fluctuations can leave observable traces on the structure of the strange quark star if the magnetize field strong enough.

Also, we may conclude that quark matter may be a source of magnetic field in the early universe. So, **the existence of magnetic fluctuations can leave observable traces on the structure of the microwave background on small angular scales, which might be detected by future satellite missions if the magnetic field is strong enough to influence the formation of large scale structure.**

**Acknowledgments** This work is supported by the Scientific and Technical Research Council of Turkey (TUBITAK) under the Grant 106T042. Also, we would like to thank to referee for their valuable suggestions.

## References

1. Grasso, D., Rubinstein, H.R.: *Phys. Lett. B* **379**, 79 (1996)
2. Cheng, B., Olinto, A.V., Schramm, D.N., Truran, J.W.: *Phys. Rev. D* **54**, 4714 (1997)
3. Zeldovich, Y.B.: *Sov. Astron.* **13**, 608 (1970)
4. Zeldovich, Y.B., Novikov, I.D.: *Relativistic astrophysics*, volume II. University of Chicago Press, Chicago (1974)
5. Adams, J., Danielson, U.H., Grasso, D., Rubinstein, H.R.: *Phys. Lett. B* **388**, 253 (1996)
6. Kosowsky, A., Loeb, A.: *Astrophys. J.* **469**, 1 (1996)
7. Barrow, J.D.: *Phys. Rev. D* **55**, 7451 (1997)
8. Barrow, J.D., Ferreira, P.G., Silk, J.: *Phys. Rev. Lett.* **78**, 3610 (1997)
9. Durrer, R., Kahniashvili, T., Yates, A.: *Phys. Rev. D* **58**, 123004 (1998)
10. Durrer, R., Ferreira, P.G., Kahniashvili, T.: *Phys. Rev. D* **61**, 043001 (2000)
11. Durrer, R.: preprint astro-ph/0609216
12. Clarkson, C.A., Coley, A.A., Maartens, R., Tsagas, C.G.: *Class. Quantum Grav.* **20**, 1519 (2003)
13. Caprini, C., Durrer, R., Kahniashvili, T.: *Phys. Rev. D* **69**, 063006 (2004)
14. Lewis, A.: *Phys. Rev. D* **70**, 043011 (2004)
15. Giovannini, M.: *Phys. Rev. D* **70**, 123507 (2004)
16. Giovannini, M.: *Phys. Rev. D* **71**, 021301 (2005)
17. Giovannini, M.: *Class. Quantum Grav.* **23**, R1 (2006)
18. Giovannini, M.: *Phys. Rev. D* **73**, 101302 (2006)
19. Subramanian, K.: preprint astro-ph/0601570
20. Yamazaki, D., Ichiki, K., Kajino, T., Mathews, G.J.: *Astrophys. J.* **646**, 719 (2006)
21. Kahniashvili, T., Ratra, B.: preprint astro-ph/0611247
22. Klein, U., Wielebinski, R., Morsi, H.W.: *Astron. Astrophys.* **190**, 41 (1988)
23. Kronberg, P.P., Perry, J.J., Zukowski, E.L.H.: *Astrophys. J.* **387**, 528 (1992)
24. Wolfe, A.M., Lanzetta, K.M., Oren, A.L.: *Astrophys. J.* **388**, 17 (1992)
25. Kronberg, P.P.: *Rep. Prog. Phys.* **57**, 325 (1994)
26. Beck, R., Brandenburg, A., Moss, D., Shukurov, A.A., Sokoloff, D.: *Ann. Rev. Astron. Astrophys.* **34**, 155 (1996)
27. Carilli, C.L., Taylor, G.B.: *Ann. Rev. Astron. Astrophys.* **40**, 391 (2004)
28. Enqvist, K.: *Int. J. Mod. Phys. D* **7**, 331 (1998)
29. Grasso, D., Rubinstein, H.: *Phys. Rep.* **348**, 163 (2001)
30. Windrow, L.M.: *Rev. Mod. Phys.* **74**, 775 (2002)
31. Giovannini, M.: preprint hep-ph/0208152
32. Giovannini, M.: *Int. J. Mod. Phys. D* **13**, 391 (2004)
33. Lugones, G., Horvath, J.E.: *Phys. Rev. D* **66**, 074017 (2002)
34. Lugones, G., Horvath, J.E.: *A & A* **403**, 173 (2003)
35. Horvath, J.E., Lugones, G.: *A & A* **422**, L1–L4 (2004)
36. Back, B.B., et al.: PHOBOS Collab. *Nucl Phys.* **A757**, 28 (2005)
37. Adams, J., et al.: STAR Collab. *Nucl Phys.* **A757**, 102 (2005)
38. Adcox, K., et al.: PHENIX Collab. *Nucl Phys.* **A757**, 184 (2005)
39. Itoh, N.: *Prog. Theo. Phys.* **44**, 291 (1970)
40. Bodmer, A.R.: *Phys. Rev.* **D4**, 1601 (1971)
41. Witten, E.: *Phys. Rev. D* **30**, 272 (1984)
42. Alcock, C., Farhi, E., Olinto, A.: *Astrophys. J.* **310**, 261 (1986)
43. Haensel, P., Zdunik, J.L., Schaeffer, R.: *Astron. Astrophys.* **160**, 121 (1986)
44. Cheng, K.S., Dai, Z.G., Lu, T.: *Int. J. Mod. Phys. D* **7**, 139 (1998)
45. Kapusta, J.: *Finite-temperature field theory*. Cambridge University Press, Cambridge (1994)
46. Sotani, H., Kohri, K., Harada, T.: *Phys. Rev.* **D69**, 084008 (2004)
47. Farhi, E., Jaffe, R.L.: *Phys. Rev. D* **30**, 2379 (1984)
48. Bannerjee, S., Ghosh, S.K., Raha, S.: *J. Phys. G Nucl. Part. Phys.* **26**, L1 (2000)
49. Dey, M., Bombaci, I., Dey, J., Ray, S., Samanta, B.C.: *Phys. Lett. B* **438**, 123 (1998); Addendum: (1999) *B* **447**, 352; Erratum: (1999) *B* **467**, 303; (1999) *Indian J. Phys.* **73B**, 377
50. Gondek-Rosińska, D., Bulik, T., Zdunik, L., Gourgoulhon, E., Ray, S., Dey, J., Dey, M.: *Astron. Astrophys.* **363**, 1005 (2000)
51. Li, X.-D., Dai, Z.-G., Wang, Z.-R.: *Astron. Astrophys.* **303**, L1 (1995)

52. Bombaci, I.: *Phys. Rev. C* **55**, 1587 (1997)
53. Li, X.-D., Bombaci, I., Dey, M., Dey, J., Heeuveld, E.P.J. van den : *Phys. Rev. Lett.* **83**, 3776 (1999)
54. Li, X.-D., Ray, S., Dey, J., Dey, M., Bombaci, I.: *Astrophys. J* **527**, L51 (1999)
55. Xu, R.X., Qiao, G.J., Zhang, B.: *Astrophys. J* **522**, L109 (1999)
56. Xu, R.X., Xu, X.B., Wu, X.J.: *Chin. Phys. Lett.* **18**, 837 (2001) astro-ph/0101013
57. Pons, J.A., Walter, F.M., Lattimer, J.M., Prakash, M., Neuhäuser, R., Penghui, A.: *Astrophys. J.* **564**, 981 (2002)
58. Fushiki, I., Gudmundsson, E.H., Pethick, C.J.: *Astrophys. J.* **342**, 958 (1989)
59. Mihara, T.A., et al.: *Nature(Lond.)* **346**, 250 (1990)
60. Chanmugam, G.: *Ann. Rev. Astron. Astrophys.* **30**, 143 (1992)
61. Kronberg, P.P.: *Rep. Prog. Phys.* **57**, 325 (1994)
62. Lai, D.: *Rev. Mod. Phys.* **73**, 629 (2001)
63. Grasso, D., Rubinstein, H.R.: *Phys. Rep.* **348**, 163 (2001)
64. Thompson, C., Duncan, R.C.: *Astrophys. J.* **473**, 322 (1996)
65. Dong, L., Shapiro, S.L.: *ApJ.* **383**, 745 (1991)
66. Shuryak, E.V.: *Phys. Lett. B* **78**, 150 (1978)
67. Kajantie, K., Miettinen, H.I., *Z. Phys. C* **9**, 341 (1981)
68. Domikos, G., Goldman, J.: *Phys. Rev D* **23**, 203 (1981)
69. Petrov, A.Z.: *Einstein spaces.* Oxford University Press, Pergamon (1969)
70. Hojman, S., Nunez, L.K., Patino, A., Rago, H.: *J. Math. Phys.* **27**, 281 (1986)
71. Green, L.H., Norris, L.K., Oliver, D.R., Davis, W.R.: *Gen. Relativ. Gravit.* **8**, 731 (1977)
72. Katzin, G.H., Levine, J., Davis, W.R.: *J. Math. Phys.* **10**, 617 (1969)
73. Zimdahl, W., Balakin, A.B.: *Phys. Rev. D* **63**, 023507 (2001)
74. Zimdahl, W., Balakin, A.B.: *Entropy* **4**, 49 (2002)
75. Zimdahl, W., Balakin, A.B., Schwarz, D.J., Pavon, D.: *Gravit. Cosmol.* **8**, 158 (2002)
76. Herrera, L., Jimenez, J.: *Phys. Rev. D* **28**, 2987 (1983)
77. Herrera, L., Poncede Leon, L.: *J. Math. Phys.* **26**, 778–784 (1985)
78. Duggal, K.L., Sharma, R.: *J. Math. Phys.* **27**, 2511 (1986)
79. Coley, A.A., Tupper, B.O.J.: *J. Math. Phys.* **30**, 2616 (1989)
80. Mason, D.P., Maartens, R.: *J. Math. Phys.* **28**, 2182 (1987)
81. Maartens, R., Mason, D.P., Tsamparlis, M.: *J. Math. Phys.* **273**, 2987 (1987)
82. Coley, A.A., Tupper, B.O.J.: *Class. Quantum Grav.* **7**, 2195 (1990)
83. Maartens, R., Maharaj, S.D., Tupper, B.O.J.: *Class. Quantum Grav.* **12**, 2577 (1995)
84. Carot, J., Coley, A.A., Sintes, A.: *Gen. Rel. Grav.* **28**, 311 (1996)
85. Yilmaz, I.: *Phys. Rev. D* **71**, 103503 (2005)
86. Yilmaz, I.: *Gen. Rel. Grav* **38**, 1397–1406 (2006)
87. Yavuz, I., Yilmaz, I., Baysal, H.: *Int. J. Mod. Phys. D* **14**, 1365–1372 (2005)
88. Mak, M.K., Harko, T.: *Int. J. Mod. Phys. D* **13**, 149 (2004)
89. Barrow, J.D., Maartens, R., Tsagas, C.G.: astro-ph/0611537 (2006)