1. $x=t^2$ and $y=t$, $0\leq t\leq 2$, so by Formula 3

$$\int_C y\,ds = \int_0^2 t\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_0^2 t\sqrt{(2t)^2+(1)^2}\,dt$$

$$= \int_0^2 t\sqrt{4t^2+1}\,dt= \frac{1}{12}(4t^2+1)^{3/2}\Big]_0^2=\frac{1}{12}(17\sqrt{17}-1)$$

2.

$$\int_C \frac{y}{x}\,ds = \int_{1/2}^1 \frac{t^3}{t^4}\sqrt{(4t^3)^2+(3t^2)^2}\,dt=\int_{1/2}^1 \frac{1}{t}\sqrt{16t^6+9t^4}\,dt=\int_{1/2}^1 t\sqrt{16t^2+9}\,dt$$

$$= \frac{1}{48}(16t^2+9)^{3/2}\Big]_{1/2}^1=\frac{1}{48}(25^{3/2}-13^{3/2})=\frac{1}{48}(125-13\sqrt{13})$$

3. Parametric equations for C are $x=4\cos t$, $y=4\sin t$, $-\frac{\pi}{2}\leq t\leq\frac{\pi}{2}$. Then

$$\int_C xy^4\,ds = \int_{-\pi/2}^{\pi/2}(4\cos t)(4\sin t)^4\sqrt{(-4\sin t)^2+(4\cos t)^2}\,dt$$

$$= \int_{-\pi/2}^{\pi/2}4^5\cos t\sin^4 t\sqrt{16(\sin^2 t+\cos^2 t)}\,dt$$

$$= 4^5\int_{-\pi/2}^{\pi/2}(\sin^4 t\cos t)(4)\,dt=(4)^6\left[\frac{1}{5}\sin^5 t\right]_{-\pi/2}^{\pi/2}=\frac{2\cdot 4^6}{5}=1638.4$$

4. Parametric equations for C are $x=1+3t$, $y=2+5t$, $0\leq t\leq 1$. Then

$$\int_C ye^x\,ds=\int_0^1(2+5t)e^{1+3t}\sqrt{3^2+5^2}\,dt=\sqrt{34}\int_0^1(2+5t)e^{1+3t}\,dt$$

Integrating by parts with $u=2+5t\Rightarrow du=5\,dt$, $dv=e^{1+3t}\Rightarrow v=\frac{1}{3}e^{1+3t}dt$ gives

$$\int_C ye^x\,ds = \sqrt{34}\left[\frac{1}{3}(2+5t)e^{1+3t}-\frac{5}{9}e^{1+3t}\right]_0^1$$

$$= \sqrt{34}\left[\left(\frac{7}{3}-\frac{5}{9}\right)e^4-\left(\frac{2}{3}-\frac{5}{9}\right)e\right]=\frac{\sqrt{34}}{9}(16e^4-e)$$

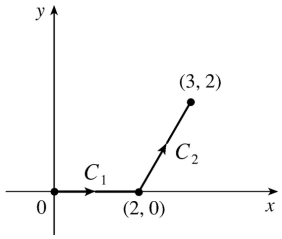
5. If we choose x as the parameter, parametric equations for C are $x=x$, $y=x^2$ for $1\le x\le 3$ and

$$\begin{aligned}\int_C (xy+\ln x)\,dy &= \int_1^3 (x\cdot x^2+\ln x)2x\,dx=\int_1^3 2(x^4+x\ln x)\,dx\\ &= 2\left[\frac{1}{5}x^5+\frac{1}{2}x^2\ln x-\frac{1}{4}x^2\right]_1^3 \text{ (by integrating by parts in the second term)}\\ &= 2\left(\frac{243}{5}+\frac{9}{2}\ln 3-\frac{9}{4}-\frac{1}{5}+\frac{1}{4}\right)=\frac{464}{5}+9\ln 3\end{aligned}$$

6. Choosing y as the parameter, we have $x=e^y$, $y=y$, $0\le y\le 1$. Then

$$\int_c xe^y\,dx=\int_0^1 e^y(e^y)e^y\,dy=\int_0^1 e^{3y}\,dy=\left.\frac{1}{3}e^{3y}\right]_0^1=\frac{1}{3}(e^3-1).$$

7.



$C=C_1+C_2$

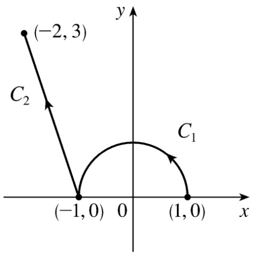
On C_1 : $x=x$, $y=0\Rightarrow dy=0\,dx$, $0\le x\le 2$.

On C_2 : $x=x$, $y=2x-4\Rightarrow dy=2\,dx$, $2\le x\le 3$.

Then

$$\begin{aligned}\int_C xy\,dx+(x-y)\,dy &=\int_{C_1} xy\,dx+(x-y)\,dy+\int_{C_2} xy\,dx+(x-y)\,dy\\ &=\int_0^2 (0+0)\,dx+\int_2^3\left[\left(2x^2-4x\right)+\left(-x+4\right)\left(2\right)\right]dx\\ &=\int_2^3 (2x^2-6x+8)\,dx=\frac{17}{3}\end{aligned}$$

8.



$C=C_1+C_2$

On C_1 : $x=\cos t\Rightarrow dx=-\sin t\,dt$, $y=\sin t\Rightarrow$
$dy=\cos t\,dt$, $0\leq t\leq \pi$.
On C_2 : $x=-1-t\Rightarrow dx=-dt$, $y=3t\Rightarrow$
$dy=3dt$, $0\leq t\leq 1$.

Then

$$\begin{aligned}
\int_C \sin x\,dx+\cos y\,dy &= \int_{C_1} \sin x\,dx+\cos y\,dy+\int_{C_2} \sin x\,dx+\cos y\,dy \\
&= \int_0^{\pi} \sin(\cos t)(-\sin t\,dt)+\cos(\sin t)\cos t\,dt \\
&\quad +\int_0^1 \sin(-1-t)(-dt)+\cos(3t)(3dt) \\
&= \Big[-\cos(\cos t)+\sin(\sin t)\Big]_0^{\pi}+\Big[-\cos(-1-t)+\sin(3t)\Big]_0^1 \\
&= -\cos(\cos\pi)+\sin(\sin\pi)+\cos(\cos 0)-\sin(\sin 0) \\
&\quad -\cos(-2)+\sin(3)+\cos(-1)-\sin(0) \\
&= -\cos(-1)+\sin 0+\cos(1)-\sin 0-\cos(-2)+\sin 3+\cos(-1) \\
&= -\cos 1+\cos 1-\cos 2+\sin 3+\cos 1=\cos 1-\cos 2+\sin 3
\end{aligned}$$

where we have used the identity $\cos(-\theta)=\cos\theta$.

9. $x=4\sin t$, $y=4\cos t$, $z=3t$, $0\leq t\leq \dfrac{\pi}{2}$. Then by Formula 9,

$$\begin{aligned}
\int_C xy^3\,ds &= \int_0^{\pi/2}(4\sin t)(4\cos t)^3\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\,dt \\
&= \int_0^{\pi/2}4^4\cos^3 t\sin t\sqrt{(4\cos t)^2+(-4\sin t)^2+(3)^2}\,dt \\
&= \int_0^{\pi/2}256\cos^3 t\sin t\sqrt{16(\cos^2 t+\sin^2 t)+9}\,dt \\
&= 1280\int_0^{\pi/2}\cos^3 t\sin t\,dt=-320\cos^4 t\Big]_0^{\pi/2}=320
\end{aligned}$$

10. Parametric equations for C are $x=4t$, $y=6-5t$, $z=-1+6t$, $0\le t\le 1$. Then

$$\int_C x^2 z\,ds = \int_0^1 (4t)^2(6t-1)\sqrt{4^2+(-5)^2+6^2}\,dt=\sqrt{77}\int_0^1(96t^3-16t^2)\,dt$$

$$=\sqrt{77}\left[96\cdot\frac{t^4}{4}-16\cdot\frac{t^3}{3}\right]_0^1=\frac{56}{3}\sqrt{77}$$

11. Parametric equations for C are $x=t$, $y=2t$, $z=3t$, $0\le t\le 1$. Then

$$\int_C xe^{yz}\,ds = \int_0^1 te^{(2t)(3t)}\sqrt{1^2+2^2+3^2}\,dt=\sqrt{14}\int_0^1 te^{6t^2}\,dt$$

$$=\sqrt{14}\left[\frac{1}{12}e^{6t^2}\right]_0^1=\frac{\sqrt{14}}{12}(e^6-1)$$

12. $\sqrt{(dx/dt)^2+(dy/dt)^2+(dz/dt)^2}=\sqrt{1^2+(2t)^2+(3t^2)^2}=\sqrt{1+4t^2+9t^4}$. Then

$$\int_C(2x+9z)\,ds=\int_0^1(2t+9t^3)\sqrt{1+4t^2+9t^4}\,dt \text{ [let } u=1+4t^2+9t^4\Rightarrow\frac{1}{4}\,du=(2t+9t^3)\,dt\text{]}$$

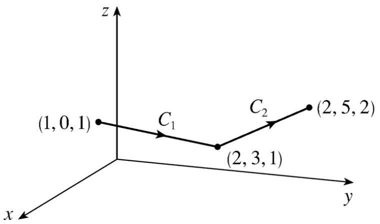
$$=\int_1^{14}\frac{1}{4}\sqrt{u}\,du=\left[\frac{1}{6}u^{3/2}\right]_1^{14}=\frac{1}{6}(14^{3/2}-1)$$

13. $\int_C x^2y\sqrt{z}\,dz=\int_0^1(t^3)^2(t)\sqrt{t^2}\cdot 2t\,dt=\int_0^1 2t^9\,dt=\left[\frac{1}{5}t^{10}\right]_0^1=\frac{1}{5}$

14.

$$\int_C z\,dx+x\,dy+y\,dz=\int_0^1 t^2\cdot 2t\,dt+t^2\cdot 3t^2\,dt+t^3\cdot 2t\,dt=\int_0^1(2t^3+5t^4)\,dt$$

$$=\left[\frac{1}{2}t^4+t^5\right]_0^1=\frac{1}{2}+1=\frac{3}{2}$$

15. 

On C_1: $x=1+t\Rightarrow dx=dt$, $y=3t\Rightarrow dy=3\,dt$, $z=1$
$\Rightarrow dz=0\,dt$, $0\le t\le 1$.

On C_2 : $x=2 \Rightarrow dx=0\,dt$, $y=3+2t \Rightarrow$
$dy=2\,dt$, $z=1+t \Rightarrow dz=dt$, $0 \leq t \leq 1$.

Then $\int_C (x+yz)\,dx+2x\,dy+xyz\,dz$

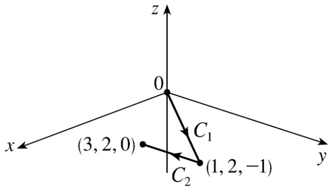
$$=\int_{C_1} (x+yz)\,dx+2x\,dy+xyz\,dz+\int_{C_2} (x+yz)\,dx+2x\,dy+xyz\,dz$$

$$=\int_0^1 (1+t+(3t)(1))\,dt+2(1+t)\cdot\ 3\,dt+(1+t)(3t)(1)\cdot\ 0\,dt$$
$$+\int_0^1 (2+(3+2t)(1+t))\cdot\ 0\,dt+2(2)\cdot\ 2\,dt+(2)(3+2t)(1+t)\,dt$$

$$=\int_0^1 (10t+7)\,dt+\int_0^1 (4t^2+10t+14)\,dt$$

$$=\Big[5t^2+7t\Big]_0^1+\left[\frac{4}{3}t^3+5t^2+14t\right]_0^1=12+\frac{61}{3}=\frac{97}{3}$$

16.



On C_1 : $x=t \Rightarrow dx=dt$, $y=2t \Rightarrow dy=2\,dt$, $z=-t$
$\Rightarrow dz=-dt$, $0 \leq t \leq 1$.
On C_2 : $x=1+2t \Rightarrow dx=2\,dt$, $y=2 \Rightarrow$
$dy=0\,dt$, $z=-1+t \Rightarrow dz=dt$, $0 \leq t \leq 1$.

Then $\int_C x^2\,dx+y^2\,dy+z^2\,dz$

$$=\int_{C_1} x^2\,dx+y^2\,dy+z^2\,dz+\int_{C_2} x^2\,dx+y^2\,dy+z^2\,dz$$

$$=\int_0^1 t^2\,dt+(2t)^2\cdot\ 2\,dt+(-t)^2(-dt)+\int_0^1 (1+2t)^2\cdot\ 2\,dt+2^2\cdot\ 0\,dt+(-1+t)^2\,dt$$

$$=\int_0^1 8t^2\,dt+\int_0^1 (9t^2+6t+3)\,dt=\left[\frac{8}{3}t^3\right]_0^1+\Big[3t^3+3t^2+3t\Big]_0^1=\frac{35}{3}$$

17. **(a)** Along the line $x=-3$, the vectors of $\boldsymbol{F}$ have positive $y-$ components, so since the path goes upward, the integrand $\boldsymbol{F}\cdot\boldsymbol{T}$ is always positive. Therefore $\int_{C_1}\boldsymbol{F}\cdot d\boldsymbol{r}=\int_{C_1}\boldsymbol{F}\cdot\boldsymbol{T}\,ds$ is positive.

(b) All of the (nonzero) field vectors along the circle with radius 3 are pointed in the clockwise direction, that is, opposite the direction to the path. So $\boldsymbol{F}\cdot\boldsymbol{T}$ is negative, and therefore $\int_{C_2}\boldsymbol{F}\cdot d\boldsymbol{r}=\int_{C_2}\boldsymbol{F}\cdot\boldsymbol{T}\,ds$ is negative.

18. Vectors starting on C_1 point in roughly the same direction as C_1, so the tangential component $\boldsymbol{F}\cdot\boldsymbol{T}$ is positive. Then $\int_{C_1}\boldsymbol{F}\cdot d\boldsymbol{r}=\int_{C_1}\boldsymbol{F}\cdot\boldsymbol{T}\,ds$ is positive. On the other hand, no vectors starting on C_2 point in the same direction as C_2, while some vectors point in roughly the opposite direction, so we would expect $\int_{C_2}\boldsymbol{F}\cdot d\boldsymbol{r}=\int_{C_2}\boldsymbol{F}\cdot\boldsymbol{T}\,ds$ to be negative.

19. $\boldsymbol{r}(t)=t^2\boldsymbol{i}-t^3\boldsymbol{j}$, so $\boldsymbol{F}(\boldsymbol{r}(t))=(t^2)^2(-t^3)^3\boldsymbol{i}-(-t^3)\sqrt{t^2}\,\boldsymbol{j}=-t^{13}\boldsymbol{i}+t^4\boldsymbol{j}$ and $\boldsymbol{r}'(t)=2t\boldsymbol{i}-3t^2\boldsymbol{j}$.
Thus $\int_C\boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^1\boldsymbol{F}(\boldsymbol{r}(t))\cdot\boldsymbol{r}'(t)dt=\int_0^1(-2t^{14}-3t^6)dt=\left[-\frac{2}{15}t^{15}-\frac{3}{7}t^7\right]_0^1=-\frac{59}{105}$.

20. $\boldsymbol{F}(\boldsymbol{r}(t))=(t^2)(t^3)\boldsymbol{i}+(t)(t^3)\boldsymbol{j}+(t)(t^2)\boldsymbol{k}=t^5\boldsymbol{i}+t^4\boldsymbol{j}+t^3\boldsymbol{k}$, $\boldsymbol{r}'(t)=\boldsymbol{i}+2t\,\boldsymbol{j}+3t^2\boldsymbol{k}$.
Thus $\int_C\boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^2\boldsymbol{F}(\boldsymbol{r}(t))\cdot\boldsymbol{r}'(t)dt=\int_0^2(t^5+2t^5+3t^5)dt=t^6\Big]_0^2=64.$

21.

$$\int_C\boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^1\left\langle\sin t^3,\cos(-t^2),\mathrm{t}^4\right\rangle\cdot\left\langle 3t^2,-2\mathrm{t},1\right\rangle dt$$

$$=\int_0^1(3t^2\sin t^3-2t\cos t^2+t^4)dt=\left[-\cos t^3-\sin t^2+\frac{1}{5}t^5\right]_0^1=\frac{6}{5}-\cos 1-\sin 1$$

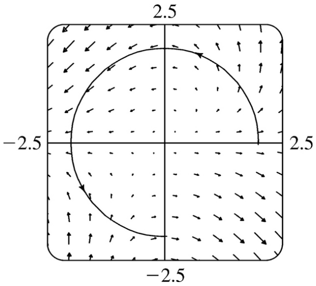
22.

$$\int_C\boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^\pi\left\langle\cos\mathrm{t},\sin\mathrm{t},-t\right\rangle\cdot\left\langle 1,\cos\mathrm{t},-\sin t\right\rangle dt=\int_0^\pi(\cos t+\sin t\cos t+t\sin t)dt$$

$$=\left[\sin t+\frac{1}{2}\sin^2 t+(\sin t-t\cos t)\right]_0^\pi=\pi$$

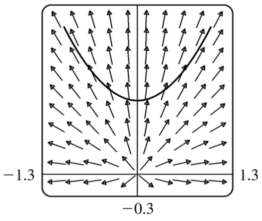
23. We graph $\mathbf{F}(x,y)=(x-y)\,\mathbf{i}+xy\,\mathbf{j}$ and the curve C. We see that most of the vectors starting on C point in roughly the same direction as C, so for these portions of C the tangential component $\mathbf{F}\cdot\mathbf{T}$ is positive. Although some vectors in the third quadrant which start on C point in roughly the opposite direction, and hence give negative tangential components, it seems reasonable that the effect of these portions of C is outweighed by the positive tangential components. Thus, we would expect $\int_C \mathbf{F}\cdot d\mathbf{r}=\int_C \mathbf{F}\cdot\mathbf{T}\,ds$ to be positive.

To verify, we evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$. The curve C can be represented by $\mathbf{r}(t)=2\cos t\,\mathbf{i}+2\sin t\,\mathbf{j}$, $0\le t\le \frac{3\pi}{2}$, so $\mathbf{F}(\mathbf{r}(t))=(2\cos t-2\sin t)\,\mathbf{i}+4\cos t\sin t\,\mathbf{j}$ and $\mathbf{r}'(t)=-2\sin t\,\mathbf{i}+2\cos t\,\mathbf{j}$. Then



$$\begin{aligned}\int_C \mathbf{F}\cdot d\mathbf{r} &= \int_0^{3\pi/2}\mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,dt\\ &= \int_0^{3\pi/2}\left[-2\sin t(2\cos t-2\sin t)+2\cos t(4\cos t\sin t)\right]dt\\ &= 4\int_0^{3\pi/2}(\sin^2 t-\sin t\cos t+2\sin t\cos^2 t)\,dt\\ &= 3\pi+\frac{2}{3}\quad\text{[using a CAS]}\end{aligned}$$

24. We graph $\mathbf{F}(x,y)=\dfrac{x}{\sqrt{x^2+y^2}}\,\mathbf{i}+\dfrac{y}{\sqrt{x^2+y^2}}\,\mathbf{j}$ and the curve C. In the first quadrant, each vector starting on C points in roughly the same direction as C, so the tangential component $\mathbf{F}\cdot\mathbf{T}$ is positive. In the second quadrant, each vector starting on C points in roughly the direction opposite to C, so $\mathbf{F}\cdot\mathbf{T}$ is negative. Here, it appears that the tangential components in the first and second quadrants



counteract each other, so it seems reasonable to guess that $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_C \boldsymbol{F}\cdot \boldsymbol{T}\,ds$ is zero. To verify, we evaluate $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$. The curve C can be represented by $\boldsymbol{r}(t)=t\,\boldsymbol{i}+(1+t^2)\boldsymbol{j}$, $-1\le t\le 1$, so

$\boldsymbol{F}(\boldsymbol{r}(t))=\dfrac{t}{\sqrt{t^2+(1+t^2)^2}}\,\boldsymbol{i}+\dfrac{1+t^2}{\sqrt{t^2+(1+t^2)^2}}\,\boldsymbol{j}$ and $\boldsymbol{r}'(t)=\boldsymbol{i}+2t\,\boldsymbol{j}$. Then

$$\begin{aligned}\int_C \boldsymbol{F}\cdot d\boldsymbol{r} &= \int_{-1}^{1}\boldsymbol{F}(\boldsymbol{r}(t))\cdot \boldsymbol{r}'(t)\,dt\\ &= \int_{-1}^{1}\left(\frac{t}{\sqrt{t^2+(1+t^2)^2}}+\frac{2t(1+t^2)}{\sqrt{t^2+(1+t^2)^2}}\right)dt\\ &= \int_{-1}^{1}\frac{t(3+2t^2)}{\sqrt{t^4+3t^2+1}}\,dt=0 \quad \text{[since the integrand is an odd function]}\end{aligned}$$

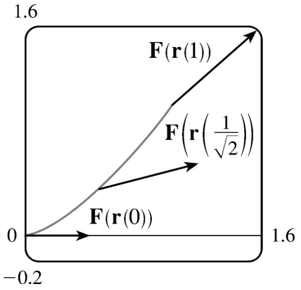
25. **(a)** $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^1\left\langle e^{t^2-1},\mathrm{t}^5\right\rangle\cdot\left\langle 2\mathrm{t},3t^2\right\rangle dt=\int_0^1\left(2te^{t^2-1}+3t^7\right)dt=\left[e^{t^2-1}+\frac{3}{8}t^8\right]_0^1=\frac{11}{8}-1/e$

(b) $\boldsymbol{r}(0)=\boldsymbol{0}$, $\boldsymbol{F}(\boldsymbol{r}(0))=\left\langle e^{-1},0\right\rangle$;

$\boldsymbol{r}\left(\frac{1}{\sqrt{2}}\right)=\left\langle\frac{1}{2},\frac{1}{2\sqrt{2}}\right\rangle$, $\boldsymbol{F}\left(\boldsymbol{r}\left(\frac{1}{\sqrt{2}}\right)\right)=\left\langle e^{-1/2},\frac{1}{4\sqrt{2}}\right\rangle$;

$\boldsymbol{r}(1)=\left\langle 1,1\right\rangle$, $\boldsymbol{F}(\boldsymbol{r}(1))=\left\langle 1,1\right\rangle$.

In order to generate the graph with Maple, we use the PLOT command (not to be confused with the plot command) to define each of the vectors. For example,

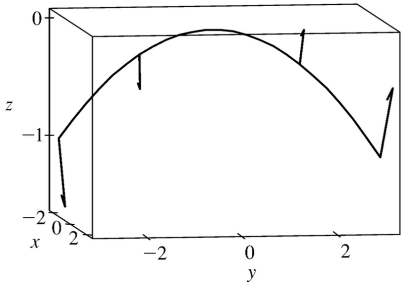


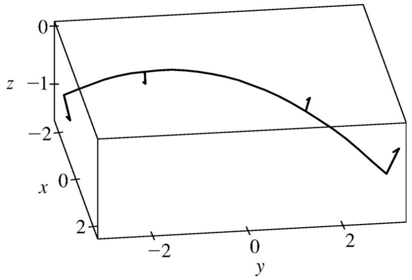
```
v1:=PLOT(CURVES( [ [0, 0], [evalf(1/exp(1)), 0] ] ) );
```

generates the vector from the vector field at the point (0, 0) (but without an arrowhead) and gives it the name v1. To show everything on the same screen, we use the display command. In Mathematica, we use ListPlot (with the PlotJoined −> True option) to generate the vectors, and then Show to show everything on the same screen.

26. **(a)** $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_{-1}^{1}\langle 2\mathrm{t}, t^2, 3t\rangle\cdot\langle 2, 3, -2t\rangle dt=\int_{-1}^{1}(4t+3t^2-6t^2)dt=\left[2t^2-t^3\right]_{-1}^{1}=-2$

(b) Now $\boldsymbol{F}(\boldsymbol{r}(t))=\langle 2t, t^2, 3t\rangle$, so $\boldsymbol{F}(\boldsymbol{r}(-1))=\langle -2, 1, -3\rangle$, $\boldsymbol{F}\left(\boldsymbol{r}\left(-\frac{1}{2}\right)\right)=\left\langle -1, \frac{1}{4}, -\frac{3}{2}\right\rangle$, $\boldsymbol{F}\left(\boldsymbol{r}\left(\frac{1}{2}\right)\right)=\left\langle 1, \frac{1}{4}, \frac{3}{2}\right\rangle$, and $\boldsymbol{F}(\boldsymbol{r}(1))=\langle 2, 1, 3\rangle$.





27. The part of the astroid that lies in the quadrant is parametrized by $x=\cos^3 t$, $y=\sin^3 t$, $0\le t\le \frac{\pi}{2}$.

Now $\frac{dx}{dt}=3\cos^2 t(-\sin t)$ and $\frac{dy}{dt}=3\sin^2 t\cos t$, so

$\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}=\sqrt{9\cos^4 t\sin^2 t+9\sin^4 t\cos^2 t}=3\cos t\sin t\sqrt{\cos^2 t+\sin^2 t}=3\cos t\sin t$.

Therefore $\int_C x^3 y^5\, ds=\int_0^{\pi/2}\cos^9 t\sin^{15} t(3\cos t\sin t)dt=\frac{945}{16{,}777{,}216}\pi$.

28. We parametrize the line as $\mathrm{r}\,(t)=\langle 1, 2, 1\rangle+t\left[\langle 6, 4, 5\rangle-\langle 1, 2, 1\rangle\right]=(1+5t)\boldsymbol{i}+(2+2t)\boldsymbol{j}+(1+4t)\boldsymbol{k}$, $0\le t\le 1$. Using a CAS, we calculate

$$\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^1\left\langle (1+5t)^4 e^{2+2t}, \ln(1+4t), \sqrt{(2+2t)^2+(1+4t)^2}\right\rangle\cdot\langle 5, 2, 4\rangle dt$$

$$=\frac{5235e^4}{4}-\frac{6285e^2}{4}+\frac{9\sqrt{5}\sinh^{-1}\left(\frac{14}{3}\right)}{25}-\frac{9\sqrt{5}\sinh^{-1}\left(\frac{4}{3}\right)}{25}+\frac{5\ln 5}{2}+\frac{14\sqrt{41}}{5}-\frac{4\sqrt{5}}{5}-2$$

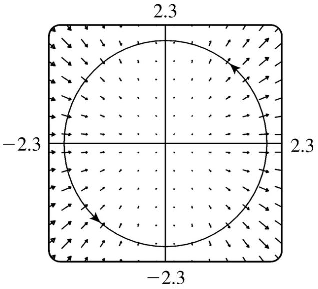
$$=\frac{5235e^4}{4}-\frac{6285e^2}{4}-\frac{18\sqrt{5}\ln 3}{25}+\frac{9\sqrt{5}\ln(14+\sqrt{205})}{25}+\frac{5\ln 5}{2}+\frac{14\sqrt{41}-4\sqrt{5}}{5}-2$$

The first answer is the one given by Maple. The two answers are equivalent by Equation 7.6.3 [ET 3.9.3].

29. A calculator or CAS gives $\int_C x\sin y\,ds=\int_1^2 \ln t\sin(e^{-t})\sqrt{(1/t)^2+(-e^{-t})^2}\,dt\approx 0.052$.

30. **(a)** We parametrize the circle C as $\boldsymbol{r}(t)=2\cos t\,\boldsymbol{i}+2\sin t\,\boldsymbol{j}$, $0\leq t\leq 2\pi$. So $\boldsymbol{F}(\boldsymbol{r}(t))=\left\langle 4\cos^2 t, 4\cos t\sin t\right\rangle$, $\boldsymbol{r}'(t)=\left\langle -2\sin t, 2\cos t\right\rangle$, and $W=\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^{2\pi}(-8\cos^2 t\sin t+8\cos^2 t\sin t)\,dt=0$.

(b)



From the graph, we see that all of the vectors in the field are perpendicular to the path. This indicates that the field does no work on the particle, since the field never pulls the particle in the direction in which it is going. In other words, at any point along C, $\boldsymbol{F}\cdot\boldsymbol{T}=0$, and so certainly $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=0$.

31. We use the parametrization $x=2\cos t$, $y=2\sin t$, $-\frac{\pi}{2}\leq t\leq\frac{\pi}{2}$. Then $ds=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\sqrt{(-2\sin t)^2+(2\cos t)^2}\,dt=2\,dt$, so $m=\int_C k\,ds=2k\int_{-\pi/2}^{\pi/2}dt=2k(\pi)$,
$\bar{x}=\frac{1}{2\pi k}\int_C xk\,ds=\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2}(2\cos t)2\,dt=\frac{1}{2\pi}\left[4\sin t\right]_{-\pi/2}^{\pi/2}=\frac{4}{\pi}$,
$\bar{y}=\frac{1}{2\pi k}\int_C yk\,ds=\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2}(2\sin t)2\,dt=0$. Hence $(\bar{x},\bar{y})=\left(\frac{4}{\pi},0\right)$.

32. We use the parametrization $x=r\cos t$, $y=r\sin t$, $0\leq t\leq\frac{\pi}{2}$. Then $ds=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\sqrt{(-r\sin t)^2+(r\cos t)^2}\,dt=r\,dt$, so
$m=\int_C (x+y)\,ds=\int_0^{\pi/2}(r\cos t+r\sin t)r\,dt=r^2\left[\sin t-\cos t\right]_0^{\pi/2}=2r^2$,

$$\overline{x} = \frac{1}{2r^2}\int_C x(x+y)\,ds = \frac{1}{2r^2}\int_0^{\pi/2}(r^2\cos^2 t + r^2\cos t\sin t)\,r\,dt = \frac{r}{2}\left[\frac{t}{2}+\frac{\sin 2t}{4}-\frac{\cos 2t}{4}\right]_0^{\pi/2}$$

$$= \frac{r(\pi+2)}{8}$$, and

$$\overline{y} = \frac{1}{2r^2}\int_C y(x+y)\,ds = \frac{1}{2r^2}\int_0^{\pi/2}(r^2\sin t\cos t + r^2\sin^2 t)\,r\,dt$$

$$= \frac{r}{2}\left[-\frac{\cos 2t}{4}+\frac{t}{2}-\frac{\sin 2t}{4}\right]_0^{\pi/2} = \frac{r(\pi+2)}{8} .$$

Therefore $(\overline{x}, \overline{y}) = \left(\frac{r(\pi+2)}{8}, \frac{r(\pi+2)}{8}\right)$.

33. **(a)** $\overline{x} = \frac{1}{m}\int_C x\rho(x, y, z)\,ds$, $\overline{y} = \frac{1}{m}\int_C y\rho(x, y, z)\,ds$, $\overline{z} = \frac{1}{m}\int_C z\rho(x, y, z)\,ds$ where $m = \int_C \rho(x, y, z)\,ds$.

(b) $m = \int_C k\,ds = k\int_0^{2\pi}\sqrt{4\sin^2 t + 4\cos^2 t + 9}\,dt = k\sqrt{13}\int_0^{2\pi} dt = 2\pi k\sqrt{13}$,

$\overline{x} = \frac{1}{2\pi k\sqrt{13}}\int_0^{2\pi} k2\sqrt{13}\sin t\,dt = 0$, $\overline{y} = \frac{1}{2\pi k\sqrt{13}}\int_0^{2\pi} k2\sqrt{13}\cos t\,dt = 0$,

$\overline{z} = \frac{1}{2\pi k\sqrt{13}}\int_0^{2\pi}(k\sqrt{13})(3t)\,dt = \frac{3}{2\pi}(2\pi^2) = 3\pi$. Hence $(\overline{x}, \overline{y}, \overline{z}) = (0, 0, 3\pi)$.

34.

$$m = \int_C (x^2+y^2+z^2)\,ds = \int_0^{2\pi}(t^2+1)\sqrt{(1)^2+(-\sin t)^2+(\cos t)^2}\,dt = \int_0^{2\pi}(t^2+1)\sqrt{2}\,dt$$

$$= \sqrt{2}\left(\frac{8}{3}\pi^3 + 2\pi\right) ,$$

$$\overline{x} = \frac{1}{\sqrt{2}\left(\frac{8}{3}\pi^3+2\pi\right)}\int_0^{2\pi}\sqrt{2}\,(t^3+t)\,dt = \frac{4\pi^4+2\pi^2}{\frac{8}{3}\pi^3+2\pi} = \frac{3\pi(2\pi^2+1)}{4\pi^2+3} ,$$

$\bar{y}=\frac{3}{2\sqrt{2}\,\pi\,(4\pi^2+3)}\int_0^{2\pi}(\sqrt{2}\cos t)(t^2+1)\,dt=0$, and

$\bar{z}=\frac{3}{2\sqrt{2}\,\pi\,(4\pi^2+3)}\int_0^{2\pi}(\sqrt{2}\sin t)(t^2+1)\,dt=0$. Hence $(\bar{x},\bar{y},\bar{z})=\left(\frac{3\pi(2\pi^2+1)}{4\pi^2+3},0,0\right)$.

35.

From Example 3, $\rho(x,y)=k(1-y)$, $x=\cos t$, $y=\sin t$, and $ds=dt$, $0\le t\le\pi\Rightarrow$

$I_x=\int_C y^2\rho(x,y)\,ds=\int_0^{\pi}\sin^2 t\,dt=k\int_0^{\pi}(\sin^2 t-\sin^3 t)\,dt$

$=\frac{1}{2}k\int_0^{\pi}(1-\cos 2t)\,dt-k\int_0^{\pi}(1-\cos^2 t)\sin t\,dt$ [Let $u=t$, $du=-\,tdt-\,3pt$ in the second integral]

$=k\left[\frac{\pi}{2}+\int_1^{-1}(1-u^2)\,du\right]=k\left(\frac{\pi}{2}-\frac{4}{3}\right)$

$I_y=\int_C x^2\rho(x,y)\,ds=k\int_0^{\pi}\cos^2 t(1-\sin t)\,dt=\frac{k}{2}\int_0^{\pi}(1+\cos 2t)\,dt-k\int_0^{\pi}\cos^2 t\sin t\,dt$

$=k\left(\frac{\pi}{2}-\frac{2}{3}\right)$, using the same substitution as above.

36.

The wire is given as $x=2\sin t$, $y=2\cos t$, $z=3t$, $0\le t\le 2\pi$ with $\rho(x,y,z)=k$. Then

$ds=\sqrt{(2\cos t)^2+(-2\sin t)^2+3^2}=\sqrt{4(\cos^2 t+\sin^2 t)+9}=\sqrt{13}$ and

$I_x=\int_C(y^2+z^2)\rho(x,y,z)\,ds=\int_0^{2\pi}(4\cos^2 t+9t^2)(k)\sqrt{13}\,dt=\sqrt{13}\,k\left[4\left(\frac{1}{2}t+\frac{1}{4}\sin 2t\right)+3t^3\right]_0^{2\pi}$

$=\sqrt{13}\,k(4\pi+24\pi^3)=4\sqrt{13}\,\pi k(1+6\pi^2)$

$I_y=\int_C(x^2+z^2)\rho(x,y,z)\,ds=\int_0^{2\pi}\left(4\sin^2 t+9t^2\right)(k)\sqrt{13}\,dt=\sqrt{13}\,k\left[4\left(\frac{1}{2}t-\frac{1}{4}\sin 2t\right)+3t^3\right]_0^{2\pi}$

$=\sqrt{13}\,k(4\pi+24\pi^3)=4\sqrt{13}\,\pi k(1+6\pi^2)$

$I_z=\int_C(x^2+y^2)\rho(x,y,z)\,ds=\int_0^{2\pi}(4\sin^2 t+4\cos^2 t)(k)\sqrt{13}\,dt=4\sqrt{13}\,k\int_0^{2\pi}dt=8\pi\sqrt{13}\,k$

37.

$$W=\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^{2\pi}\left\langle t-\sin t,\,3-\cos t\right\rangle\cdot\left\langle 1-\cos t,\,\sin t\right\rangle dt$$

$$=\int_0^{2\pi}(t-t\cos t-\sin t+\sin t\cos t+3\sin t-\sin t\cos t)\,dt$$

$$=\int_0^{2\pi}(t-t\cos t+2\sin t)\,dt=\left[\frac{1}{2}t^2-(t\sin t+\cos t)-2\cos t\right]_0^{2\pi}$$ [by integrating by parts in the second term]

$$=2\pi^2$$

38.

$x=x$, $y=x^2$, $-1\le x\le 2$,

$$W=\int_{-1}^{2}\left\langle x\sin x^2,\,\mathrm{x}^2\right\rangle\cdot\left\langle 1,\,2x\right\rangle dx=\int_{-1}^{2}(x\sin x^2+2x^3)\,dx=\left[-\frac{1}{2}\cos x^2+\frac{1}{2}x^4\right]_{-1}^{2}$$

$$=\frac{1}{2}(15+\cos 1-\cos 4)$$

39.

$\boldsymbol{r}(t)=\left\langle 1+2t,\,4t,\,2t\right\rangle$, $0\le t\le 1$,

$$W=\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^1\left\langle 6t,\,1+4t,\,1+6t\right\rangle\cdot\left\langle 2,\,4,\,2\right\rangle dt=\int_0^1(12t+4(1+4t)+2(1+6t))\,dt$$

$$=\int_0^1(40t+6)\,dt=\left[20t^2+6t\right]_0^1=26$$

40.

$\boldsymbol{r}(t)=2\,\boldsymbol{i}+t\,\boldsymbol{j}+5t\,\boldsymbol{k}$, $0\le t\le 1$. Therefore

$$W=\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^1\frac{K\left\langle 2,\,t,\,5t\right\rangle}{(4+26t^2)^{3/2}}\cdot\left\langle 0,\,1,\,5\right\rangle dt=K\int_0^1\frac{26t}{(4+26t^2)^{3/2}}\,dt$$

$$=K\left[-(4+26t^2)^{-1/2}\right]_0^1=K\left(\frac{1}{2}-\frac{1}{\sqrt{30}}\right)$$

41.

Let $\boldsymbol{F}=185\boldsymbol{k}$. To parametrize the staircase, let

$x=20\cos t$, $y=20\sin t$, $z=\frac{90}{6\pi}t=\frac{15}{\pi}t$, $0\leq t\leq 6\pi \Rightarrow$

$$W=\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^{6\pi}\langle 0,0,185\rangle\cdot\left\langle -20\sin t, 20\cos t, \frac{15}{\pi}\right\rangle dt=(185)\frac{15}{\pi}\int_0^{6\pi}dt=(185)(90)$$

$$\approx 1.67\times 10^4 \text{ ft}-\text{lb}$$

42.

This time m is a function of t : $m=185-\frac{9}{6\pi}t=185-\frac{3}{2\pi}t$. So let $\boldsymbol{F}=\left(185-\frac{3}{2\pi}t\right)\boldsymbol{k}$. To parametrize the staircase, let $x=20\cos t$, $y=20\sin t$, $z=\frac{90}{6\pi}t=\frac{15}{\pi}t$, $0\leq t\leq 6\pi$. Therefore

$$W=\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^{6\pi}\left\langle 0,0,185-\frac{3}{2\pi}t\right\rangle\cdot\left\langle -20\sin t, 20\cos t, \frac{15}{\pi}\right\rangle dt=\frac{15}{\pi}\int_0^{6\pi}\left(185-\frac{3}{2\pi}t\right)dt$$

$$=\frac{15}{\pi}\left[185t-\frac{3}{4\pi}t^2\right]_0^{6\pi}=90\left(185-\frac{9}{2}\right)\approx 1.62\times 10^4 \text{ ft}-\text{lb}$$

43. **(a)** $\boldsymbol{r}(t)=\langle\cos t,\sin t\rangle$, $0\leq t\leq 2\pi$, and let $\boldsymbol{F}=\langle a,b\rangle$. Then

$$W=\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^{2\pi}\langle a,b\rangle\cdot\langle -\sin t,\cos t\rangle dt$$

$$=\int_0^{2\pi}(-a\sin t+b\cos t)dt=\left[a\cos t+b\sin t\right]_0^{2\pi}$$

$$=a+0-a+0=0$$

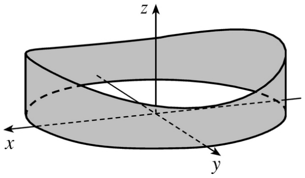
(b) Yes. $\boldsymbol{F}(x,y)=k\,\boldsymbol{x}=\langle kx,ky\rangle$ and

$$W=\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_0^{2\pi}\langle k\cos t,k\sin t\rangle\cdot\langle -\sin t,\cos t\rangle dt$$

$$=\int_0^{2\pi}(-k\sin t\cos t+k\sin t\cos t)dt=\int_0^{2\pi}0\,dt=0$$

44. Consider the base of the fence in the $xy-$ plane, centered at the origin, with the height given by $z=h(x,y)$. The fence can be graphed using the parametric equations
$x=10\cos u$, $y=10\sin u$,

$$\begin{aligned} z &= v\left[4+0.01((10\cos u)^2-(10\sin u)^2)\right] \\ &= v(4+\cos^2 u-\sin^2 u) \\ &= v(4+\cos 2u),\ 0\leq u\leq 2\pi,\ 0\leq v\leq 1. \end{aligned}$$



The area of the fence is $\int_C h(x,y)\,ds$ where C , the base of the fence, is given by $x=10\cos t$, $y=10\sin t$, $0\leq t\leq 2\pi$. Then

$$\begin{aligned} \int_C h(x,y)\,ds &= \int_0^{2\pi}\left[4+0.01((10\cos t)^2-(10\sin t)^2)\right]\sqrt{(-10\sin t)^2+(10\cos t)^2}\,dt \\ &= \int_0^{2\pi}(4+\cos 2t)\sqrt{100}\,dt=10\left[4t+\frac{1}{2}\sin 2t\right]_0^{2\pi} \\ &= 10(8\pi)=80\pi\,\text{m}^2 \end{aligned}$$

If we paint both sides of the fence, the total surface area to cover is $160\pi\ \text{m}^2$, and since 1 L of paint covers $100\ \text{m}^2$, we require $\frac{160\pi}{100}=1.6\pi\approx 5.03$ L of paint.

45. The work done in moving the object is $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}=\int_C \boldsymbol{F}\cdot\boldsymbol{T}\,ds$. We can approximate this integral by dividing C into 7 segments of equal length $\Delta s=2$ and approximating $\boldsymbol{F}\cdot\boldsymbol{T}$, that is, the tangential component of force, at a point (x_i^*, y_i^*) on each segment. Since C is composed of straight line segments, $\boldsymbol{F}\cdot\boldsymbol{T}$ is the scalar projection of each force vector onto C. If we choose (x_i^*, y_i^*) to be the point on the segment closest to the origin, then the work done is $\int_C \boldsymbol{F}\cdot\boldsymbol{T}\,ds\approx\sum_{i=1}^{7}\left[\boldsymbol{F}(x_i^*, y_i^*)\cdot\boldsymbol{T}(x_i^*, y_i^*)\right]\Delta s=[2+2+2+2+1+1+1](2)=22$. Thus, we estimate the work done to be approximately 22 J.

46. Use the orientation pictured in the figure. Then since $\boldsymbol{B}$ is tangent to any circle that lies in the plane perpendicular
to the wire, $\boldsymbol{B}=|\boldsymbol{B}|\boldsymbol{T}$ where $\boldsymbol{T}$ is the unit tangent to the circle C : $x=r\cos\theta$, $y=r\sin\theta$. Thus
$\boldsymbol{B}=|\boldsymbol{B}|\left\langle -\sin\theta, \cos\theta\right\rangle$. Then
$\int_C \boldsymbol{B}\cdot d\boldsymbol{r}=\int_0^{2\pi}|\boldsymbol{B}|\left\langle -\sin\theta, \cos\theta\right\rangle\cdot\left\langle -r\sin\theta, r\cos\theta\right\rangle d\theta=\int_0^{2\pi}|\boldsymbol{B}|r\,d\theta=2\pi r|\boldsymbol{B}|$. (Note that $|\boldsymbol{B}|$ here is the magnitude of the field at a distance r from the wire's center). But by Ampere's Law
$\int_C \boldsymbol{B}\cdot d\boldsymbol{r}=\mu_0 I$. Hence $|\boldsymbol{B}|=\dfrac{\mu_0 I}{2\pi r}$.