Honors Math B
Quiz 2 Practice Questions

Throughout, F is an arbitrary field.

1. Prove that if A is a symmetric invertible matrix with entries in F, then its inverse A^{-1} is also symmetric.

2. Recall that, given a matrix $A \in M_{n\times n}(F)$, we defined $\mathrm{tr}(A) = \sum_{i=1}^{n} A_{ii}$. Prove that if $A, B \in M_{n\times n}(F)$, then $\mathrm{tr}(AB) = \mathrm{tr}(BA)$.

3. If A and B are symmetric square matrices, is AB a symmetric matrix? Provide a proof or counterexample.

4. Calculate the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}.$$

and determine an eigenvector belonging to each. Is this matrix diagonalizable over the real numbers? Why or why not?

5. Let $n > 0$ be an integer and V the vector space of real polynomials of degree less than or equal to n. Calculate the characteristic polynomial of the differentiation operator $D : V \to V$.

6. Let $V = M_{n\times n}(\mathbf{R})$, the vector space of real 2×2 matrices. Show that $\langle A, B\rangle := \mathrm{tr}(A^{\mathrm{t}}B)$ is an inner product on V. Does this work over $\mathbf{C}$? If not, what do you need to change to make it work?

7. Prove that if V is a complex inner product space and $T : V \to V$ is a linear map satisfying $\langle Tz, z\rangle = 0$ for all $z \in V$, then $T = 0$. [HINT: Plug in $z = x + y$, then $z = x + iy$.] Give an example to show that this is not true for real inner product spaces.

8. Apostol, Volume II, p. 119 exercise 9.

9. Apostol, Volume II, p. 126 exercise 12.

10. Apostol, Volume II, p. 141 exercise 12.

11. If $A \in M_{n\times n}(\mathbf{C})$, show that $\det(A) = \overline{\det(A^*)}$.