Honors Math B
Homework 10

A

Read Apostol, Volume II, pp. 257-275. Also have a look at pp. 303-313, which discuss extrema of scalar-valued functions of multiple variables, and pp. 319-322, which prove multidimensional versions of two theorems that should be familiar from the one-variable case.

B

To turn in, do Apostol p. 256 exercises 20 and 22, p. 263 exercise 10, p. 269 exercise 12, and p. 281 exercises 2 and 3.

To do for yourself, do Apostol p. 256 exercises 4, 7, 8, 9, 10, 12, and 21 and pp. 262-263 exercises 1, 2, and 7.

C

1. To turn in: A function $F : \mathbf{R}^n \to \mathbf{R}$ is *homogeneous [of degree 1]* if $F(tx) = tF(x)$ for all nonzero $t \in \mathbf{R}$ and all nonzero $x \in \mathbf{R}^n$. Suppose that F is homogeneous and continuous.
a) Prove that $F(0) = 0$.
b) Show that the directional derivative of F at 0 along y exists for all $y \in \mathbf{R}^n$.

2. To turn in: Suppose $F : \mathbf{R}^n \to \mathbf{R}$ satisfies $|F(x)| \leq c||x||^2$ for some $c \in \mathbf{R}$ and all $x \in \mathbf{R}^n$. Compute $F'(0; y)$ for all $y \in \mathbf{R}^n$.

3. To turn in: Suppose that $F : \mathbf{R}^n \to \mathbf{R}^m$ is linear. Show that the derivative of F at any $x \in \mathbf{R}^n$ is just F itself.

4. To turn in: Suppose that $G : \mathbf{R}^n \to \mathbf{R}$ is homogeneous and continuous. Prove that G is differentiable if and only if it is linear.

5. To do for yourself: Suppose $F : U \to \mathbf{R}^m$, U open in $\mathbf{R}^n$, is written in components as $F = (F_1, F_2, \ldots, F_m)$, where $F_i : U \to \mathbf{R}$ for $1 \leq i \leq m$. For any point $x \in U$, show that F is differentiable at x if and only if each F_i is differentiable at x, $1 \leq i \leq m$.