A

Read Apostol, Volume II, pp. 257-275. Also have a look at pp. 303-313, which discuss extrema of scalar-valued functions of multiple variables, and pp. 319-322, which prove multidimensional versions of two theorems that should be familiar from the one-variable case.

B

To turn in, do Apostol p. 263 exercise 10, p. 269 exercise 12, p. 276 exercises 3ab and 14, and pp. 281-82 exercises 1 and 12a.

To do for yourself, do Apostol pp. 262-63 exercises 1, 2, 7, and 11 and p. 276 exercises 6, 8, 9, and 15.

C

1. To turn in: Suppose that $F : \mathbb{R}^n \to \mathbb{R}^m$ is linear. Show that the derivative of $F$ at any $x \in \mathbb{R}^n$ is just $F$ itself.

2. To turn in: Suppose that $G : \mathbb{R}^n \to \mathbb{R}$ is homogeneous (see the last problem set for this notion) and continuous. Prove that $G$ is differentiable if and only if it is linear.

3. To turn in: Suppose $F : U \to \mathbb{R}^m$, $U$ open in $\mathbb{R}^n$, is written in coordinates as $F = (F_1, F_2, \ldots, F_m)$, where $F_i : U \to \mathbb{R}$ for $1 \leq i \leq m$. For any point $x \in U$, show that $F$ is differentiable at $x$ if and only if each $F_i$ is differentiable at $x$, $1 \leq i \leq m$.

4. To turn in: Use the previous problem to prove that if $F : U \to \mathbb{R}^m$ is a function such that all entries of its Jacobian matrix (i.e., all possible first partial derivatives) exist and are continuous, then $F$ is differentiable.