Honors Math B
Homework 4

A

Get ahead by starting to read Apostol, Volume II, Chapter 3.

B

To turn in, do the following problems in Apostol, Volume II: p. 68 exercises 11 and 12 and pp. 68-69 ("Miscellaneous exercises on matrices") exercises 1, 2abcf, and 7bde. Don't use determinants in the preceeding answers, and do note that one can take the definition of the transpose A^t can be taken to be $A^t_{ij} = A_{ji}$. Apostol uses "nonsingular" and "singular" for "invertible" and "not invertible."

To do for yourself, do p. 68 exercises 7, 8, 9, and 15 and pp. 68-69 exercises 2de, 5, and 7ac.

C

1. To turn in: A nontrivial calculation! Use Gauss-Jordan elimination to find the set of all real solutions to the following suspiciously similar-looking linear systems:

$$a)\begin{cases} 2x_1+4x_2+8x_3+6x_4 &= 0,\\ 5x_1+6x_2+8x_3+7x_4 &= 0,\\ 6x_1+7x_2+9x_3+8x_4 &= 0,\\ 5x_1+4x_2+2x_3+3x_4 &= 0. \end{cases} \quad b)\begin{cases} 2x_1+4x_2+8x_3+6x_4 &= 2,\\ 5x_1+6x_2+8x_3+7x_4 &= 9,\\ 6x_1+7x_2+9x_3+8x_4 &= 11,\\ 5x_1+4x_2+2x_3+3x_4 &= 11. \end{cases} \quad c)\begin{cases} 2x_1+4x_2+8x_3+6x_4 &= 2,\\ 5x_1+6x_2+8x_3+7x_4 &= 9,\\ 6x_1+7x_2+9x_3+8x_4 &= 6,\\ 5x_1+4x_2+2x_3+3x_4 &= 11. \end{cases}$$

2. To turn in: A square matrix A is *symmetric* if $A = A^t$ and *skew-symmetric* if $A = -A^t$.
a) Prove that if A is symmetric and invertible, then its inverse is symmetric. Give a 2×2 example.
b) Prove that if A is skew-symmetric and invertible, then its inverse is skew-symmetric. Give a 2×2 example.

3. To do for yourself: A square matrix A is *upper-triangular* if $A_{ij} = 0$ whenever $i > j$. Prove that if A is upper-triangular and invertible, then its inverse is upper-triangular. Give a 2×2 example.

4. To turn in: If A and B are $n \times n$ matrices, prove that $\operatorname{rank}(AB) \leq \min(\operatorname{rank} A, \operatorname{rank} B)$.