Honors Math B
Homework 10

A

Read Apostol, Volume II, Chapter 10 (you can skip §10.19).

B

To turn in, do Apostol p. 328 exercise 7, p. 337 exercise 5a and pp. 345-6 exercises 3, 9, 14, and 18.

To do for yourself, do Apostol p. 328 exercises 2 and 8, p. 331 exercise 1, p. 337 exercises 6 and 7, and p. 345 exercise 11.

C

1. To turn in: Suppose that $F : \mathbb{R}^2 \setminus \{0\} \to \mathbb{R}^2$ is the vector field

   $F(x, y) = \left( \frac{x + y}{x^2 + y^2}, \frac{y - x}{x^2 + y^2} \right)$.

   Show that $D_1 F_2(x, y) = D_2 F_1(x, y)$ for all points in the domain of $F$, but $F$ is not conservative.

2. To turn in: Suppose that $F : \mathbb{R}^n \setminus \{0\} \to \mathbb{R}^n$ is a vector field that can be expressed as $F(x) = f(||x||) \frac{x}{||x||}$, where $f : (0, \infty) \to \mathbb{R}$ is a continuously differentiable function (this is more or less the general form of a “radial vector field”). Show that $F$ is conservative.

3. To turn in: Consider the vector field on the open first quadrant $(0, \infty) \times (0, \infty) \subset \mathbb{R}^2$ defined by

   $F(x, y) = \left( \frac{y + 1}{x^2 y}, \frac{x + 1}{x y^2} \right)$.

   Is it conservative? Why or why not? If so, what are all the possible potentials?