Honors Math B
Homework 1

A

Read pp. 445-467 and pp. 471-482 in Apostol, Volume I for a treatment of some linear algebra concepts in $\mathbf{R}^n$ (which Apostol calls V_n for some reason). Then read pp. 551-560 and pp. 578-585 for a more general context (equivalently, pp. 3-13 and pp. 31-38 in Apostol, Volume II).

B

To turn in, do the following problems in Apostol, Volume I: exercises 6 and 7 on p. 451, exercises 4 and 19 on p. 456, and exercises 9, 10, 11, 21, and 29 on pp. 555-6. (We have not discussed the dot product and norm on $\mathbf{R}^n$ yet in class; we'll return to them later in a more general setting.)

To do for yourself, do exercises 5 and 11a on p. 450-1, exercises 2, 3, 10, and 20 on p. 456, exercises 5 and 9 on p. 477, and every other exercise on pp. 555-6 (they're all pretty short).

C

1. To turn in: Let F be a field. Prove that a function $f : U \to V$ between F-vector spaces is linear if and only if for each $n \in \mathbf{Z}_{>0}$ and for all $X_1, \ldots, X_n \in U$ and $c_1, \ldots, c_n \in F$, we have

$$f\left(\sum_{i=1}^{n} c_i X_i\right) = \sum_{i=1}^{n} c_i f(X_i).$$

2. To turn in: Let F be a field and let $f : U \to V$ be a linear map between F-vector spaces. Prove that f is injective if and only if $\ker f$ is the zero subspace (i.e., the subspace containing only the zero element.). [Recall that Apostol calls it a null space rather than a kernel.]

3. To do for yourself: Let F be a field and U, V be F-vector spaces. Prove that the set $\mathcal{L}(U, V)$ of linear maps from U to V is a subspace of the vector space $\mathcal{F}(U, V)$ of all functions from U to V.

4. To turn in: Prove that the composition of two linear maps is linear.

5. To turn in:
a) If $g : V \to W$ is a fixed linear map, prove that the map $L_g : \mathcal{L}(U, V) \to \mathcal{L}(U, W)$ given by $L_g(f) = g \circ f$ is linear. [We say "composition on the left is linear."]
b) If $f : U \to V$ is a fixed linear map, prove that the map $R_f : \mathcal{L}(V, W) \to \mathcal{L}(U, W)$ given by $R_f(g) = g \circ f$ is linear. [We say "composition on the right is linear."]