Honors Math A
Homework 9

A

Read Apostol pp. 174-194.

B

To turn in, do the following problems in Apostol: p. 180 exercise 19bcd and pp. 186-187 exercises 7, 8b, and 9.

To do for yourself, do Apostol p. 180 exercise 16 and p. 186 exercises 2 and 4.

C

1. To turn in:
a) If $f + g$ is differentiable at x, are f and g necessarily differentiable at x? Give a proof or counterexample.
b) If fg and f are both differentiable at x, what condition(s) on f imply that g is also differentiable at x? Prove it!

2. To turn in:
a) Suppose that $f(x) = xg(x)$, where g is a function on $\mathbf{R}$ which is continuous at 0. Prove that f is differentiable at 0 and find $f'(0)$ in terms of g.
b) Suppose that f is differentiable at 0 and $f(0) = 0$. Prove that $f(x) = xg(x)$ for some function g which is continuous at 0.

3. To turn in: Prove that if $|f|$ is differentiable at x and f is continuous at x, then f is also differentiable at x. [Hint: it suffices to consider only x with $f(x) = 0$ (why?); in this case, what must $|f|'(x)$ be?]

4. To do for yourself: Suppose that $f'(x) \geq M > 0$ for all $x \in [0, 1]$. Show that there is an interval of length 1/4 on which $|f| \geq M/4$.