Honors Math A
Homework 8

A

Read Apostol pp. 156-173.

B

To turn in, do the following problems in Apostol: p. 155 exercise 8, p. 168 exercises 22 and 24, and p. 174 exercise 15 (give a proof or counterexample in each case).

To do for yourself, do Apostol p. 155 exercise 4, p. 167-168 exercises 1, 10, and 38, and p. 173 exercise 9.

C

1. To turn in: Let $f : [a, b] \to \mathbf{R}$ be integrable. Prove that there is some $c \in [a, b]$ such that

$$\int_a^c f(x)\ \mathrm{d}x = \frac{1}{2} \int_a^b f(x)\ \mathrm{d}x.$$

2. To turn in: Suppose that f is continuous on $[0, 1]$ and $f(0) = f(1)$. Let $n \in \mathbf{Z}_{>0}$. Prove that there is some number $x \in [0, 1 - 1/n]$ such that $f(x) = f(x + 1/n)$. [HINT: Consider the function $g(x) = f(x) - f(x + 1/n)$.]

3. To do for yourself: (Challenge problem!) Let $0 < \alpha < 1$ be a real number such that $\alpha \neq \frac{1}{n}$ for all n. Find a function $f : [0, 1] \to \mathbf{R}$ that is continuous and satisfies $f(0) = f(1)$ but which does not satisfy $f(x) = f(x + \alpha)$ for any $x \in [0, 1 - \alpha]$.