Honors Math A
Homework 5

A

Read pp. 64-104 in Apostol.

B

To turn in, do the following problems in Apostol: pp. 70-71 exercises 3, 5, 11, and 15, and p. 84 exercise 25. For exercise 5, you may assume that every nonnegative real number has a unique nonnegative square root, as proved in Theorem I.35. For exercise 11, give a brief sketch of a proof or a counterexample for each statement.

To do for yourself, do pp. 70-71 exercises 1, 10, 16, and 17, and p. 84 exercise 27.

C

1. To turn in: What is $\sup\left\{\frac{n-1}{n} \mid n \in \mathbf{Z}_{>0}\right\}$? What is $\inf\left\{\frac{n+1}{n} \mid n \in \mathbf{Z}_{>0}\right\}$? Prove that your answers are correct.

2. To turn in: Prove the reflection property of integration: if f is integrable on $[a, b]$, then

$$\int_{-b}^{-a} f(-x)\ \mathrm{d}x = \int_a^b f(x)\ \mathrm{d}x.$$

[Hint: first prove it for step functions.]

3. To turn in: Prove that if f and $|f|$ are both integrable on $[a, b]$, then

$$\left|\int_a^b f(x)\ \mathrm{d}x\right| \leq \int_a^b |f(x)|\ \mathrm{d}x.$$

Notice the similarity to the triangle inequality!

4. To do for yourself: Is it possible for $|f|$ to be integrable when f is not? Give a proof or counterexample.

5. To turn in: A function $f : \mathbf{R} \to \mathbf{R}$ is said to be *even* if $f(-x) = f(x)$ for all x, and *odd* if $f(-x) = -f(x)$ for all x.
a) Prove that if f is both odd and even, then $f(x) = 0$ for all x.
b) Suppose that f is integrable on every closed interval $[a, b]$ and let $g(x) = \int_0^x f(t)\ \mathrm{d}t$. Prove that

if f is odd, then g is even, and that if f is even, then g is odd.

6. To turn in: Prove that $f : [a, b] \to \mathbf{R}$ is integrable if and only if for all $\epsilon > 0$, there exist step functions $s, t : [a, b] \to \mathbf{R}$ such that $s \leq f \leq t$ and $\int_a^b (t - s)(x)\,\mathrm{d}x < \epsilon$.