Honors Math A
Homework 4

A

Read the handout on cardinality, posted on the course webpage, and Apostol pp. 48-63.

B

To turn in, do the following problems in Apostol: p. 28 exercises 1, 3, 4, and 6 and p. 64 exercises 4bd and 5.

To do for yourself: p. 28 exercises 7, 10, and 11 and p. 64 exercise 2.

C

1. To turn in: Suppose $S \subseteq \mathbf{R}$ and $c \in \mathbf{R}$. Let $cS = \{cx \mid x \in S\}$ (i.e. we are "stretching" S by a factor of c).
a) Show that if $c > 0$ and S is bounded above, then cS is bounded above.
b) Show that if $c > 0$, then $\sup(cS) = c\sup(S)$, in the sense that either both sides do not exist or both sides do exist and are equal.
c) Give an example where $c < 0$ and $\sup(cS) \neq c\sup(S)$.

2. To turn in: Suppose $S \subseteq \mathbf{R}$ and $t \in \mathbf{R}$. Show that $t = \sup(S)$ if and only if both of the following are true: a) t is an upper bound for S, and b) for all $\epsilon > 0$, there exists an $x \in S$ such that $x > t - \epsilon$.

3. To turn in: Suppose that $S, T \subseteq \mathbf{R}$, both S and T are nonempty and bounded above, and there is a bijective function $f : S \to T$ such that $x \geq f(x)$ for all $x \in S$. Show that $\sup(S) \geq \sup(T)$. Can you say more if you know $x > f(x)$ for all $x \in S$?