Honors Math A
Homework 3

A

Read the remainder of the Introduction in Apostol.

B

To turn in, do the following problems in Apostol: p. 19 exercise 4, p. 21 exercises 1 (but only prove Theorem I.24) and 10, pp. 35-36 exercises 1(c) and 4, and p. 43 exercise 1(j).

To do for yourself: p. 19 exercise 3, p. 21 exercises 2, 5, and 9, p. 35 exercise 1(a)(b), and p. 43 exercises 1(b)(d)(f)(g).

C

1. To turn in: Let $S \subseteq \mathbf{Z}_{\geq 0}$ be a subset of the natural numbers. An element $m \in S$ is called a *least element* or *minimum* if for all $n \in S$, $m \leq n$.
a) Prove by induction that for all $k \in \mathbf{Z}_{\geq 0}$, the set $\{\ell \in \mathbf{Z}_{\geq 0} : \ell \leq k\}$ either contains a least element for S or does not contain any elements of S.
b) Use a) to prove the *well-ordering principle*: every nonempty subset of $\mathbf{Z}_{\geq 0}$ has a least element.

2. To turn in: A set S is said to be *finite* if there exists a bijection $f : S \to \{i \in \mathbf{Z} \mid 0 < i \leq n\} = \{1, 2, \ldots, n\}$ for some $n \in \mathbf{Z}_{\geq 0}$. Prove rigorously that the Cartesian product of two finite sets is finite. [HINT: First consider the special case $S = \{1, \ldots, m\}$ and $T = \{1, \ldots, n\}$ by induction on m. Then deduce the general case.]

3. To do for yourself: Prove that the union and intersection of two finite sets are finite. (You might want to use the well-ordering principle.)

4. To turn in: Prove that for all real $\epsilon > 0$ and all $x \in \mathbf{R}$, $|x| < \epsilon$ if and only if $-\epsilon < x < \epsilon$.