Honors Math A
Homework 2

A

Continue to read the Introduction in Apostol through p. 25, as well as §§1.2-1.3 (pp. 50-54).

B

1. To turn in: Prove that if A and B are sets, then their power sets satisfy $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$. Give an example to show that equality need not hold in general.

2. To turn in: Prove that if two sets A and B are not equal, then $\mathcal{P}(A)$ and $\mathcal{P}(B)$ are not equal.

3. To turn in: Use the definition of the product of a pair of sets to prove that if A, B, and C are sets, then $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

4. To turn in: If $f : S \to T$ and $g : T \to U$ are injective functions, prove that the composite $g \circ f$ is injective.

5. To turn in: Suppose that $f : S \to T$ and $g : T \to S$ are functions such that $g \circ f = \mathrm{id}_S : S \to S$. (We say in this case that g is a *left inverse* to f.) For each of the following statements, give a proof if true or a counterexample if false:
a) f is injective.
b) f is surjective.
c) g is injective.
d) g is surjective.

6. To turn in: Use the description of a function as its graph and some of our basic set-theory axioms to prove that if A and B are sets, then the collection of functions $f : A \to B$ is a set. This set is commonly denoted B^A.

7. To turn in: If A is a set with n elements and B is a set with m elements, how many elements does B^A have? (Since we haven't defined set cardinality yet, you don't need to offer a proof – just an explanation of why your answer is correct.)

8. To turn in: Let A be a set and let $B = \{0, 1\}$. Let $W : B^A \to \mathcal{P}(A)$ be the function that takes a function $f : A \to B$ to $f^{-1}(\{1\})$ (i.e., the subset of elements of A that get mapped to 1 via f). Prove that W is a bijection. This explains why you sometimes see the notation 2^A used to denote the power set of A – here "2" is standing in for a set with two elements.