Algebraic Topology
Homework 8

Reading associated with this assignment: Hatcher, pp. 110-131. You may use the equivalence of singular homology and ∆-homology in this assignment whenever it is convenient (we will prove it on Tuesday).


5. Carefully define the homotopy category of abelian groups, $\mathbf{K}(\mathbf{Ab})$, which as explained in class is roughly “the category of chain complexes modulo chain homotopy” in the same way that $\mathbf{hTop}$ is “the category of topological spaces modulo homotopy.” Prove any properties of chain homotopies that you use.

6. A complex of abelian groups is called acyclic if its homology groups all vanish. (Similarly, a topological space is called acyclic if its associated singular chain complex is acyclic; i.e., if its singular homology groups all vanish.) A complex of abelian groups $A$ is called contractible if the identity map on $A$ is chain homotopic to the zero map. Prove that all contractible complexes are acyclic, and give an example of an acyclic complex that is not contractible.