Recall the category \( \text{Ch}(\text{Ab}) \) of complexes of abelian groups discussed in class.

4. Prove that a morphism \( f : A \to B \) in \( \text{Ch}(\text{Ab}) \) is an isomorphism (i.e., has a two-sided inverse) if and only if each \( f_n : A_n \to B_n \) is an isomorphism of groups.

5. Suppose that \( A \) is a complex with all boundary maps equal to the zero map. Prove that \( H_n(A) \cong A_n \).

6. Given two morphisms \( f, g : A \to B \) in \( \text{Ch}(\text{Ab}) \), we can form the sum \( f + g : A \to B \) simply by letting \( (f + g)_n = f_n + g_n \). Prove that \( H_n : \text{Ch}(\text{Ab}) \to \text{Ab} \) is additive in the sense that \( H_n(f + g) = H_n(f) + H_n(g) \).

7. We say that \( A' \) is a subcomplex of a complex \( A \) if each \( A'_n \) is a subgroup of \( A_n \) and each boundary map on \( A' \) is the restriction of the corresponding boundary map of \( A \). Carefully define the kernel and image of a morphism \( f : A \to B \) of complexes, and prove the “first isomorphism theorem” of complexes: if \( f : A \to B \) is a morphism of complexes, then there is an isomorphism of complexes \( A/ \ker(f) \cong \text{im}(f) \).