

Practice Midterm II

Calculus III

March 25, 2009

1. Find the (unique) point of intersection between the two space curves defined by $r_1(t) = \langle t, e^t, e^t + 1 \rangle$ for $t \in \mathbb{R}$ and $r_2(t) = \langle 2\ln(t), t + 2, 3t - 1 \rangle$.

2. Let $r(t) = \langle t, 2t\sqrt{2t}/3, t^2/2 \rangle$ for $t \in [2, 5]$. Find the total length of the curve described by that vector function.

3. Find the curvature, the tangential and normal components of acceleration of $r(t) = \langle t, 2t^2, t^3 \rangle$ at $t = 2$ and $t = 3$.

4. An object has acceleration $a(t) = \langle e^{-2t}, 3t, t^2 \rangle$ for $t \in \mathbb{R}$. We have $v(0) = \langle 1, 2, 3 \rangle$ and $r(2) = \langle 4, 1, 5 \rangle$. Find $r(1)$.

5. Let $r(t)$ be a vector function describing the motion of an object in space, and $T(t)$, $N(t)$, $B(t)$ the unit tangent, normal and binormal vector. Show that $B'(t)$ is orthogonal to $B(t)$ and to $T(t)$.

6. Find the curvature of the plane curve $\langle t, \sin t \rangle$. When is the curvature maximal? minimal? find the center of the osculating circle to this curve at $(\pi/2, 1)$.