

Practice Final

Calculus III

April 25, 2007

1. Find the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{3x^2+y^4}$ or prove that it does not exist. Same question with $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y}{3x^2+y^4}$.
2. Find the domain and the partial derivatives of $f(x, y) = (3x^4 + 2y^2)^{1/4}$. Same question for $f(x, y, z) = \frac{z}{x^2-y}$.
3. Find all second order partial derivatives of $f(x, y, z) = x^3 - y^2z^2 + xy$.
4. Find the equation of the tangent planes of the surface $x^2 - 2y^2 + 4z = 0$ at $(4, 2, 1)$.
5. If $z = y + f(x^2 - y^2)$ and f is differentiable show that $y \frac{dz}{dx} + x \frac{dz}{dy} = x$.
6. Let $f(x, y, z) = xy^2z^3$. Find the gradient of f at $(1, 1, 1)$. When is the directional derivative along a unit vector u at $(1, 1, 1)$ equal to 0?
7. Find the local maximum, minimum and the saddle point at the function $f(x, y) = (xy)e^{x/2}$.
8. Find the minimum and maximum values of $f(x, y, z) = xy^2z^3$ on the closed ball of center at the origin and radius 5.
9. Find the minimum and maximum values of $f(x, y) = \frac{1}{x} + \frac{1}{y}$ subject to the constraint $\frac{1}{x^2} + \frac{1}{y^2} = 1$.