Mathematics W4051y
Basic Topology

Practice Midterm #1

This is an actual midterm from a previous version of the course.

Do all six problems. On each page of your blue book, write the number of the problem you work on inside a *circle*. Each problem is worth 10 points. You may use any results from lectures, homework, or Munkres (except the one being asked for!), but be sure to refer to them clearly and state the source. In grading the midterm, I will emphasize precision, concision, and clarity. Read directions carefully. Take a minute to think before beginning to write each answer. Good luck.

1. Let $X = \mathbf{Z}$. For each integer $k \geq 0$, let $A_k = \{2^k n \,|\, n \in \mathbf{Z}\}$, and let

$$\mathcal{T} = \{A_k \,|\, k \geq 0\} \cup \{\emptyset\}.$$

(a) Prove that $\mathcal{T}$ is a topology. Equipped with this topology, is X (b) Hausdorff, (c) metrizable, (d) connected? Give *brief* reasons.

2. Let X and Y be topological spaces, and let $X \times Y$ have the product topology. Let $\pi : X \times Y \to X$ be defined by $\pi(x, y) = x$. Prove that if W is open in $X \times Y$, then $\pi(W)$ is open in X.

3. Recall that the *cofinite topology* on a set X consists of $\emptyset$ together with the complements of all finite sets. Let X be a set with at least 2 points; prove that it is infinite if and only if it is connected in the cofinite topology.

4. (a) In a complete sentence, give a clear and precise definition of a *metric space* X.

(b) Prove that any metric space is Hausdorff.

5. (a) If $f : X \to Y$ is any continuous function, and $U \subset Y$ is any subset, show that

$$f^{-1}(\overline{U}) \supset \overline{f^{-1}(U)}.$$

(b) Give an example where the reverse inclusion does *not* hold.

6. Show that continuous functions take *path* connected sets to *path* connected sets, i.e., $f : X \to Y$ continuous and $A \subset X$ path connected implies $f(A)$ path connected.