Mathematics W4061y
Differentiable Manifolds

Midterm Examination
March 12, 2014

PART I: Statements and definitions (10 pts each).

1. Define (a) an *k-tensor* on a vector space V; (b) an *alternating k-tensor.*

2. State the Contraction Mapping Theorem.

3. Define the *tangent space* to a manifold $M \subset \mathbf{R}^n$ at a point p.

PART II: Proofs and examples (15 pts each).

4. Let V be an n-dimensional vector space and suppose $0 \neq \omega \in \Lambda^{n-1}V^*$. Prove there exists $\mu \in \Lambda^1 V^*$ such that $\omega \wedge \mu \neq 0$.

5. If M and P are nonempty 7-dimensional manifolds, N is a 5-dimensional manifold, and $f : M \to P$ satisfies $f = g \circ h$ where $M \xrightarrow{h} N \xrightarrow{g} P$, can f be a diffeomorphism? Either prove that it cannot or give an example where it is.

6. If f and g are integrable on a closed rectangle A, and if $f(x) \leq g(x)$ for all $x \in A$, use our definition of the integral to prove that $\int_A f \leq \int_A g$.

7. Let $U = \{(x, y) \in \mathbf{R}^2 \,|\, x > 0,\ y > 0\}$ be the open first quadrant. For a smooth function g, let S be the set of points in U satisfying $r = g(\theta)$ in polar coordinates. Prove that S is a manifold of dimension 1.