In Spivak, do the following problems: 3–10, 3–18, 3–31, 3–33.

Also do the following:

1. Let $A \subset \mathbb{R}^p$ be a closed rectangle, let $f : A \to \mathbb{R}^m$ be continuous (where $m > 0$), and let $\Gamma = \{(f(x), x) \in \mathbb{R}^{m+p} \mid x \in A\}$ be the graph of $f$. Prove that $\Gamma$ has content zero.

2. Prove that compact manifolds have content zero. Hint: use the graph description.

3. Let $S \subset \mathbb{R}^n$ be a bounded subset such that $S = \{x \in \mathbb{R}^n \mid f(x) \leq c\}$ for some differentiable $f : \mathbb{R}^n \to \mathbb{R}$ and $c \in \mathbb{R}$ such that $D_x f \neq 0$ whenever $f(x) = c$. Prove that any continuous $g : S \to \mathbb{R}$ is integrable on $S$.

4. Describe a solid torus (i.e., the set of points on or inside a torus in $\mathbb{R}^3$) as a set $S$ of the type above.