Mathematics W4081y
Differentiable Manifolds

Assignment #7

Due March 31, 2014

In Spivak, do the following problems: **3–10**, **3–18**, **3–31**, **3–33**.

Also do the following:

1. Let $A \subset \mathbf{R}^p$ be a closed rectangle, let $f : A \to \mathbf{R}^m$ be continuous (where $m > 0$), and let $\Gamma = \{(f(x), x) \in \mathbf{R}^{m+p} \,|\, x \in A\}$ be the graph of f. Prove that Γ has content zero.

2. Prove that compact manifolds have content zero. Hint: use the graph description.

3. Let $S \subset \mathbf{R}^n$ be a bounded subset such that $S = \{x \in \mathbf{R}^n \,|\, f(x) \leq c\}$ for some differentiable $f : \mathbf{R}^n \to \mathbf{R}$ and $c \in \mathbf{R}$ such that $D_x f \neq 0$ whenever $f(x) = c$. Prove that any continuous $g : S \to \mathbf{R}$ is integrable on S.

4. Describe a solid torus (i.e., the set of points on or inside a torus in $\mathbf{R}^3$) as a set S of the type above.