Mathematics W4081y
Differentiable Manifolds

Assignment #4
Due February 24, 2014


Also do the following:

1. An affine transformation $A : \mathbb{R}^n \to \mathbb{R}^m$ is a function of the form $A(u) = v_0 + L(u)$, where $v_0 \in \mathbb{R}^n$ is fixed and $L : \mathbb{R}^n \to \mathbb{R}^m$ is linear.
   (a) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by
       $$f \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^3 + 2xy + y^2 \\ x^2 + y \end{pmatrix}.$$  
       What is the affine transformation best approximating $f^{-1}$ near $f \left( \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right)$?
   (b) Given $x \in \mathbb{R}$, for what values of $y \in \mathbb{R}$ does $f$ have a smooth inverse near $f \left( \begin{pmatrix} x \\ y \end{pmatrix} \right)$?