Mathematics W4081y
Differentiable Manifolds

Assignment #4
Due February 24, 2014

In Spivak, do the following problems: **2–28**bd, **2–29**, **2–30**, **2–34**, **2–37**, **2–38**, **2–39**.

Also do the following:

1. An *affine transformation* $A : \mathbf{R}^n \to \mathbf{R}^m$ is a function of the form $A(u) = v_0 + L(u)$, where $v_0 \in \mathbf{R}^n$ is fixed and $L : \mathbf{R}^n \to \mathbf{R}^m$ is linear.

   (a) Let $f : \mathbf{R}^2 \to \mathbf{R}^2$ be given by

$$f \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^3 + 2xy + y^2 \\ x^2 + y \end{pmatrix}.$$

   What is the affine transformation best approximating f^{-1} near $f\begin{pmatrix} 1 \\ 1 \end{pmatrix}$?

   (b) Given $x \in \mathbf{R}$, for what values of $y \in \mathbf{R}$ does f have a smooth inverse near $f\begin{pmatrix} x \\ y \end{pmatrix}$?