

Mathematics V2010y

Linear Algebra

Assignment #2

Due January 31, 2007

From the text, read section §1.5 and do the Practice Problems.

Then do the following exercises.

From 1.2 in the text, do 1, 5, 17, 33, and 39–43.

From 1.5 in the text, do 14 and 20.

Also do the following.

1. Compute (a) $\mathbf{u} + \mathbf{v}$, (b) $\mathbf{u} + \mathbf{v} + \mathbf{w}$, (c) $2\mathbf{u} + 2\mathbf{v} + \mathbf{w}$, and (d) $-\mathbf{u} - 3\mathbf{v} + \mathbf{w}$ when $\mathbf{u} = (1, 2, 3)$, $\mathbf{v} = (-3, 1, -2)$, and $\mathbf{w} = (2, -3, -1)$.
2. Let $\mathbf{u} = (\sqrt{3}/2, 1/2)$ and $\mathbf{v} = (-1/2, \sqrt{3}/2) \in \mathbf{R}^2$. Compute $\mathbf{u} \cdot \mathbf{u}$, $\mathbf{v} \cdot \mathbf{v}$, $\mathbf{u} \cdot \mathbf{v}$, and $\mathbf{v} \cdot \mathbf{u}$. Sketch \mathbf{u} and \mathbf{v} . What angle does \mathbf{u} make with the x -axis? What about \mathbf{v} ?
3. Two nonzero vectors are said to be *perpendicular* if their dot product is zero; they are said to be *parallel* if one is a scalar multiple of the other.
 - (a) Find two vectors in \mathbf{R}^3 perpendicular to $(1, 1, 1)$ and to each other.
 - (b) If \mathbf{u} and \mathbf{v} in \mathbf{R}^2 are both perpendicular to a nonzero vector \mathbf{w} , are they parallel to each other? Why or why not?
 - (c) What about the same question in \mathbf{R}^3 ? See (a) above.
 - (d) What about the same question in \mathbf{R}^4 ?
 - (e) What about the same question in \mathbf{R}^n for $n > 4$?
4. (a) At a racetrack, you can place three different bets on a horse named Echelon. A *win* bet is payable only if Echelon comes in first; the odds are 12:1. A *place* bet is payable if he comes in first or second; the odds are 9:1. A *show* bet is payable if he comes in first, second or third; the odds are 5:1. A bookie offers you a bet that costs \$15 and pays \$15 if Echelon comes in third, \$60 if he comes in second, and \$120 if he comes in first. You like the payoff but distrust the bookie. Is there a way to get the same payoff at the track by betting $\$x$ on win, $\$y$ on place, and $\$z$ on show? How much will it cost you total? Hint: write down a 3×3 linear system expressing the payoffs in terms of x, y, z . Solve it with row operations and back substitution.
 - (b) Another bookie offers you a \$10 *exacta* bet: it pays \$90 only if Echelon comes exactly in third place. Can you duplicate this by betting on win, place and show? Why or why not? What is the solution or solutions of the linear system?

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5. The diagram below is a top view of the beams in a ceiling. All beams weigh 10 kg/m , and all are 3 m long except for the two shorter ones, which are 2 m long. At the T-joints, the base of the T rests on the crosspiece. All T-joints are 1 m from one end of the crosspiece.
- Argue that because of symmetry, the 6 T-joints fall into 3 pairs of joints which bear the same weight. Label the pairs in the diagram.
 - Write down a system of 3 equations in 3 unknowns involving the weights at the 3 pairs of joints.
 - Solve the linear system by row operations and back substitution, to compute the weight on the joints in kg. Hint: the denominators get nasty. It may be convenient to use operation II to clear the denominators from time to time.

