In the text, read sections §5.1 and §5.2 and do the Practice Problems.

Then find (a) the eigenvalues and (b) a basis for each eigenspace of the following matrices.

\[
\begin{pmatrix}
8 & 2 \\
12 & -2
\end{pmatrix}
\begin{pmatrix}
1 & 1 \\
-1 & 3
\end{pmatrix}
\begin{pmatrix}
-3 & 0 & 6 \\
-6 & 3 & 6 \\
-3 & 0 & 6
\end{pmatrix}
\begin{pmatrix}
-3 & 1 & -6 \\
1 & 1 & 2 \\
-1 & -5 & -2
\end{pmatrix}
\begin{pmatrix}
6 & 3 & 5 \\
9 & 8 & 9 \\
-13 & -9 & -12
\end{pmatrix}
\begin{pmatrix}
2 & 1 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & -5 & 0 \\
0 & 0 & 0 & 0 & -5
\end{pmatrix}
\]

Also do the following exercises.

1. What are the eigenvalues of an upper-triangular matrix?

2. Let \( A \) be an invertible matrix with eigenvalues \( \lambda_1, \ldots, \lambda_k \).
   
   (a) Can any of these be 0? Why or why not?
   
   (b) Show that every eigenvector of \( A \) is also an eigenvector of \( A^{-1} \). What is its eigenvalue?
   
   (c) What are the eigenvalues of \( A^{-1} \) in terms of \( \lambda_1, \ldots, \lambda_k \)? Why?

3. Let \( v \) be an eigenvector of \( A \) with eigenvalue \( \lambda \). Use induction to show that, for any \( k \geq 0 \), \( v \) is also an eigenvector of \( A^k \) with eigenvalue \( \lambda^k \).

4. Suppose that a square matrix satisfies \( A^2 = A \). What possible eigenvalues can it have? Why? Give some examples.

Also do the following exercises from the text.
From §5.1, do 1, 42, 45, and 46.
From §5.2, do 1, 2, 41, 46, 49, 50, and 51.