Mathematics V1202
Calculus IV

Midterm Examination #2

November 20, 2006
2:40–3:55 pm

PART I: True/False with reason. If true, say so and give a *brief* reason (1 sentence). If false, say so and give a *specific* example where it fails. 6 points each.

1. If C is the unit circle and $\mathbf{F} = (f(x), g(y))$, where f and g are functions of one variable with continuous derivatives, then $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$.

2. The line integral of any constant vector field of unit length along a smooth curve C equals the arclength of C.

3. If $\mathbf{F}(x, y, z) = (2x, 2y, 2z)$ and S is a closed surface oriented outward, then $\int\int_S \mathbf{F} \cdot d\mathbf{S} \geq 0$.

PART II: Long answers. Give reasons and show all work. 13 points each.

4. For $\mathbf{F} = (y + \sin^3 x)\mathbf{i} + (2yz + \cos^3 y)\mathbf{j} + x^2\mathbf{k}$, evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve parametrized by $\mathbf{r}(t) = (\sin t, \cos t, \sin 2t)$. Hint: C lies on the surface $z = 2xy$.

5. Let $\mathbf{v} = (a, b, c)$ be any constant vector. If $f(x, y, z)$ is any function with continuous second partials, show that $\nabla \cdot (\mathbf{v} \times \nabla f) = 0$.

6. Find a nonzero vector field $\mathbf{F}$ in the plane such that $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ whenever the endpoints of C both lie on the parabola $y = x^2$. Hint: use the fundamental theorem of line integrals.

7. Use the change of variables $x = 2u + 3v$, $y = 3u - 2v$ to evaluate $\iint_R (x + y)\, dA$, where R is the square with vertices $(0, 0)$, $(2, 3)$, $(5, 1)$, and $(3, -2)$. Hint: $u = (2x + 3y)/13$, $v = (3x - 2y)/13$.