

Mathematics V1208y
Honors Mathematics B

Midterm Examination

March 10, 2010

READ AND FOLLOW CAREFULLY ALL INSTRUCTIONS BELOW

Turn off all electronic devices.

Brief visits to the men's or women's room (just out the door to your left) are OK, but one at a time only, and you must tell Andrew your name before you go.

Write your name, "Honors Math B, Prof. Thaddeus," and the number of blue books on the cover of each blue book.

Write your name on the attendance sheet when it comes around.

Write all answers and work in your blue books. Do not hand in this sheet.

On each page you use, write the number of the problem *in a circle in the margin*.

You may do more than one problem on a single page. Just put all numbers by their problems.

You may do the problems out of order, but this is discouraged as it can lead to misgrading.

When there is any doubt, state briefly but clearly what statements from Apostol, lecture, or assignments you are using.

In grading the exams, I will emphasize accuracy, brevity, and clarity. Aim for all three.

Attempt all 7 problems. Each is worth 10 points. Good luck!

1. State the definition of reduced row-echelon form.

2. (a) Diagonalize $A = \begin{pmatrix} -4 & 6 \\ -3 & 5 \end{pmatrix}$.

(b) Give an explicit formula, depending on k , for the upper right-hand entry of A^k .

3. (a) Find the inverse of the matrix $B = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 0 & 1 \\ 2 & 0 & 1 \end{pmatrix}$.

(b) If $\vec{c} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$, find all solutions \vec{x} of the inhomogeneous system $B\vec{x} = \vec{c}$.

4. Let C be the $n \times n$ matrix such that $C_{ij} = i + j$.

Use row operations to show that $\det C = 0$ if and only if $n > 2$.

5. Let D be a square matrix such that $D^2 = D$.

Prove that the eigenvalues of D are all either 0 or 1.

6. If $E \in M_{nn}$ has distinct eigenvalues $\lambda_1, \dots, \lambda_n$, prove that $\det E = \lambda_1 \lambda_2 \cdots \lambda_n$.

7. We saw in class that $A_0 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is not diagonalizable. Prove, however, that:

(a) $A_t = \begin{pmatrix} 1 & 1 \\ t & 1 \end{pmatrix}$ is diagonalizable over \mathbb{R} if t is real and positive;

(b) $A_t = \begin{pmatrix} 1 & 1 \\ t & 1 \end{pmatrix}$ is diagonalizable over \mathbb{C} , but not \mathbb{R} , if t is real and negative.

You needn't actually diagonalize A_t .