Linear Algebra

THE GRAM-SCHMIDT ORTHONORMALIZATION PROCESS

1. Start with any basis $\vec{u}_1, \ldots, \vec{u}_k$ for a subspace $S \subset \mathbb{R}^n$.

2. Let $\vec{v}_1 = \vec{u}_1$.

3. Let $\vec{v}_2 = \vec{u}_2 - \dfrac{\vec{u}_2 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1} \vec{v}_1$.

4. Let $\vec{v}_3 = \vec{u}_3 - \dfrac{\vec{u}_3 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1} \vec{v}_1 - \dfrac{\vec{u}_3 \cdot \vec{v}_2}{\vec{v}_2 \cdot \vec{v}_2} \vec{v}_2$.

5. For each succeeding i, let

$$\vec{v}_i = \vec{u}_i - \sum_{j=1}^{i-1} \frac{\vec{u}_i \cdot \vec{v}_j}{\vec{v}_j \cdot \vec{v}_j} \vec{v}_j .$$

6. Finally, let $\vec{w}_i = \dfrac{1}{\|\vec{v}_i\|} \vec{v}_i$ for each i.

Then $\vec{v}_1, \ldots, \vec{v}_k$ is an orthogonal basis for S and $\vec{w}_1, \ldots, \vec{w}_k$ is an orthonormal basis for S.