Mathematics V1207x
Honors Mathematics A

SOME NOTES ON THE MIDTERM EXAM

(1) Read carefully the comments in red ink and be sure you understand them.

(2) If asked a question about a natural number (or a finite set), consider induction.

(3) If asked to show that something does NOT exist, consider contradiction. Don't make contradiction a reflex, however: a direct argument, if it exists, may be simpler and shorter.

(4) When your arguments involve a supremum, infimum, limit, or integral, there is, in principle, an implicit obligation to prove that the object in question exists. Just as, when you divide by some quantity in a proof, there is an implicit obligation to prove that it is nonzero. Of course, if the reasons for the existence are really obvious, it's better to say nothing.

(5) A statement like $\lim_{x \to n^2+1} f(x) = k$ is meaningful, but a statement like $\lim_{x^2+1 \to n} f(x) = k$ is not. Similarly, a statement beginning $\forall\, x \in \mathbb{R}$ is meaningful, but a statement beginning $\forall\, x^2 + 1 \in \mathbb{R}$ is not.

(6) The integral of $f : [a, b] \to \mathbb{R}$ is NOT $\sum_{i=1}^{n} f(x_i)(x_i - x_{i-1})$.

(7) The definition of a *finite* set does NOT involve the existence of upper or lower bounds for that set. For a set like $\{\text{apple, banana}\}$, whose elements are not real numbers, an upper bound is not even meaningful.

(8) The very fact that a problem is on an exam is, in itself, a big hint. It means that a succinct solution to the problem exists. If your solution is convoluted, think for a moment before writing. Perhaps it can be simplified.